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Some properties of generalized rough sets

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ABSTRACT

Some algebraic and topological properties of generalized rough sets, associated with generalized lower and upper approximations are studied. Generalized definable sets give rise to two topological spaces. Degree of accuracy (DAG) for generalized rough sets induces two types of equivalence relations, one is defined on X, whereas the other is defined on $P(Y)$. Both of these relations strictly manage an order in classes of X and $P(Y)$. It has been shown that DAG induces two types of fuzzy subsets: one of the set X and the other of $P(Y)$. Finally, some properties of these fuzzy sets have been discussed.

1. Introduction

Theory of rough sets, proposed by Pawlak, has emerged as a new mathematical tool to deal with uncertainty. It is very handy in data mining and decision-making. Since its inception, it is getting popularity among the researchers, working in various areas. In this theory, equivalence relations play a vital role to define lower and upper approximations of a subset of the universe set. Equivalence classes of the universe set, obtained by an equivalence relation are the fundamental building blocks for defining lower and upper approximations of a subset of the universe set. A subset of the universe set is called definable, if it can be written as union of some equivalence classes of the universe set, otherwise it is not definable. In general a subset of the universe is not definable, however, it can be approximated by two definable subsets of the universe. One is called the lower approximation of the subset and it is the union of all equivalence classes contained in the subset. The other is called the upper approximation of the subset, it is the union of all equivalence classes of the universe set which have non-empty intersection with the subset. The lower approximation of a subset is the greatest definable subset of the universe contained in the subset whereas the upper approximation is the least definable subset containing the subset. Rough set theory is the mathematical tool to deal with vague concepts. This theory has been applied successfully to many areas such as knowledge discovery, machine learning, data analysis, approximate classification and conflict analysis [22,23]. Applications of this theory can also be seen in [11,12,14,27].

Although, rough set theory, proposed by Pawlak has been applied successfully in various areas, yet there are difficulties as well. These difficulties may be due to imprecise human knowledge about the objects under consideration. The basic requirement of this concept is an equivalence relation among the objects under consideration. Some time due to incomplete

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information such an equivalence relation is not easy to find. Therefore, some more general models have been proposed. Bonikowski et al. introduced the concept of rough sets with covering [2]. Properties and applications of rough sets with covering have been studied in [20,29,30]. Fuzzy rough sets were defined by Dubois and Prade [7]. Yao [32] introduced the concept of generalized rough sets based on relations. These rough sets have been investigated in [13,16,35]. Feng et al. proposed another type of rough sets, in which lower and upper approximations of a subset are obtained with the help of soft sets [8], also, see [5,6,9,10,15,18,21,23–26,28,33,36].

In order to have a more flexible tool for analysis of an information system recently, Davvaz has studied the concept of generalized rough sets called by him as T-rough sets [4]. This is another generalization of rough sets. In this type of generalized rough sets, instead of equivalence relations we require set valued maps. This technique is useful, where it is difficult to find an equivalence relation among the elements of the universe set. In generalized rough sets, a set valued map gives rise to two operators, called lower and upper generalized approximation operators. These are actually interior and closure operators respectively. In this paper we study the topologies of those sets which are fixed by these operators.

The basic purpose to measure the degree of accuracy for rough sets is to excavate numerical characters to analyze the data in an effective way. Different types of accuracy and roughness measures for rough set theories have been defined and studied as per their contexts. Banerjee and Pal introduced the concept of roughness measure of fuzzy sets [1]. Measure of roughness in rough sets with covering has been studied in [30]. Liu and Zhu studied roughness measurement in generalized rough sets based on relations [16]. Chakrabarty et al. studied the measure of fuzziness in rough sets [3]. Pawlak explained [23] that accuracy measure is very useful to provide an idea about the accuracy of information related to an equivalence relation for a particular classification. We extend this idea for generalized rough sets and study it in more depth. It is very important to measure the degree of accuracy for generalized rough sets (DAGs) for their practical application, such as pattern recognition, image processing and decision-making. In an information system for decision-making problems, DAG helps us to arrange decision attributes in a certain order of preference for a certain selection of objects. On the other hand for a particular decision attribute, DAG provides an order among possible selections and helps us to find the optimal/minimal choice. This method is direct, efficient and a very small number of calculations are required for it.

This paper is arranged in the following manner: In Section 2, some basic concepts about rough sets introduced by Pawlak and generalized rough sets have been given. Concept of topology determined by generalized definable sets is studied in Section 3. It is shown that two topologies are determined by generalized rough sets, one is on the set *Y* while the other is on the set *X*. In Section 4, concept of degree of accuracy for generalized rough sets is studied. Some basic properties of DAG have been studied there. It is shown that DAG induces two equivalence relations one on the set *X* and the other on the set *P*(*Y*). Equivalence classes obtained in this manner maintain a strict order among themselves. In Section 5, some properties of two types of fuzzy sets induced by DAG have been studied. With the help of an example, it has been demonstrated, how DAG can be helpful in decision-making problems.

2. Preliminaries

In the theory of rough sets, presented by Pawlak, equivalence relations are very important. Equivalence classes are basic building blocks for lower and upper approximations of a subset of the universe set.

Let *U* be a non-empty finite set called the universe set. Let σ be an equivalence relation on *U*. Then, (U,σ) is called an approximation space. If *X* is a subset of *U*, then *X* may or may not be written as union of the equivalence classes of *U*. If *X* can be written as union of some equivalence classes of *U*, then we say *X* is definable otherwise it is not definable. If *X* is not definable, then we can approximate it by two definable subsets called lower and upper approximations of *X* as the following:

$$\underline{app}(X) = \bigcup\{[x]_\sigma : [x]_\sigma \subseteq X\},$$
$$\overline{app}(X) = \bigcup\{[x]_\sigma : [x]_\sigma \cap X \neq \emptyset\}.$$

A rough set is the pair $(\underline{app}(X), \overline{app}(X))$. The set $\overline{app}(X) - \underline{app}(X)$ is called boundary region. Clearly, if $\underline{app}(X) = \overline{app}(X)$ Then, *X* is definable and $\overline{app}(X) - \underline{app}(X)$ is an empty set.

It can be seen that for a set X, $\underline{app}(X)$ is the greatest definable set contained in *X*, whereas $\overline{app}(X)$ is the least definable set containing *X*. The following well known proposition is given by Pawlak.

Proposition 1 22. *If σ is an equivalence relation on a set U, and X, Y are subsets of U then the following hold*

1. $\underline{app}(X) \subseteq X \subseteq \overline{app}(X)$,
2. $\underline{app}(X \cap Y) = \underline{app}(X) \cap \underline{app}(Y)$,
3. $X \subseteq Y \Rightarrow \overline{app}(X) \subseteq \overline{app}(Y)$,
4. $X \subseteq Y \Rightarrow \underline{app}(X) \subseteq \underline{app}(Y)$,
5. $\underline{app}(X \cup Y) \supseteq \underline{app}(X) \cup \underline{app}(Y)$,
6. $\overline{app}(X \cup Y) = \overline{app}(X) \cup \overline{app}(Y)$,
7. $\overline{app}(X \cap Y) \subseteq \overline{app}(X) \cap \overline{app}(Y)$.