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Nondeterministic automata: Equivalence, bisimulations, and uniform relations ☆

Miroslav Ćirić *, Jelena Ignjatović, Milan Bašić, Ivana Jančić

University of Niš, Faculty of Sciences and Mathematics, Višegradska 33, 18000 Niš, Serbia

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ABSTRACT

In this paper we study the equivalence of nondeterministic automata pairing the concept of a bisimulation with the recently introduced concept of a uniform relation. In this symbiosis, uniform relations serve as equivalence relations which relate states of two possibly different nondeterministic automata, and bisimulations ensure compatibility with the transitions, initial and terminal states of these automata. We define six types of bisimulations, but due to the duality we discuss three of them: forward, backward–forward, and weak forward bisimulations. For each of these three types of bisimulations we provide a procedure which decides whether there is a bisimulation of this type between two automata, and when it exists, the same procedure computes the greatest one. We also show that there is a uniform forward bisimulation between two automata if and only if the factor automata with respect to the greatest forward bisimulation equivalences on these automata are isomorphic. We prove a similar theorem for weak forward bisimulations, using the concept of a weak forward isomorphism instead of an isomorphism. We also give examples that explain the relationships between the considered types of bisimulations.

1. Introduction

One of the most important problems of automata theory is to determine whether two given automata are equivalent, which usually means to determine whether their behavior is identical. In the context of deterministic or nondeterministic automata the behavior of an automaton is understood to be the language that is recognized by it, and two automata are considered *equivalent*, or more precisely *language-equivalent*, if they recognize the same language. For deterministic finite automata the equivalence problem is solvable in polynomial time, but for nondeterministic finite automata it is computationally hard (PSPACE-complete [24,54,56]). Another important issue is to express the language-equivalence of two automata as a relation between their states, if such relationship exists, or find some kind of relations between states which would imply the language-equivalence. The language-equivalence of two deterministic automata can be expressed in terms of relationships between their states, but in the case of nondeterministic automata the problem is more complicated.

A widely-used notion of "equivalence" between states of automata is that of *bisimulation*. Bisimulations have been introduced in computer science by Milner [43] and Park [47], where they have been used to model equivalence between various systems, as well as to reduce the number of states of these systems. Roughly at the same time they have been also discovered in some areas of mathematics, e.g., in modal logic and set theory. They are employed today in a many areas of computer

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* Corresponding author. Tel.: +381 18224492; fax: +381 18533014.
E-mail addresses: miroslav.ciric@pmf.edu.rs (M. Ćirić), jekaignjatovic73@gmail.com (J. Ignjatović), basic_milan@yahoo.com (M. Bašić), ivanajancic84@gmail.com (I. Jančić).

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science, such as functional languages, object-oriented languages, types, data types, domains, databases, compiler optimizations, program analysis, and verification tools. For more information about bisimulations we refer to [1,13,21,25,42,44,45,49,53].

The most common structures on which bisimulations have been studied are labeled transition systems, i.e., labeled directed graphs, which are essentially nondeterministic automata without fixed initial and terminal states. A definition of bisimulations for nondeterministic automata that takes into account initial and terminal states was given by Kozen in [39]. In numerous papers dealing with bisimulations mostly one type of bisimulations has been studied, called just bisimulation, like in the Kozen's book [39], or strong bisimulations, like in [44,45,49]. In this paper we differentiate two types of simulations, forward and backward simulations. Considering that there are four cases when a relation R and its inverse R^{-1} are forward or backward simulations, we distinguish four types of bisimulations. We define two homotypic bisimulations, forward and backward bisimulations, where both R and R^{-1} are forward or backward simulations, and two heterotypic bisimulations, backward–forward and forward–backward bisimulations, where R is a backward and R^{-1} a forward simulation or vice versa. Distinction between forward and backward simulations, and forward and backward bisimulations, has been also made, for instance, in [9,26,42] (for various kinds of automata), but more or less these concepts differ from the concepts having the same name which are considered here. More similar to our concepts of forward and backward simulations and bisimulations are those studied in [8], and in [27,28] (for tree automata).

It is worth noting that forward and backward bisimulations, and backward–forward and forward–backward bisimulations, are dual concepts, i.e., backward and forward–backward bisimulations on a nondeterministic automaton are forward and backward–forward bisimulations on its reverse automaton. This means that for any universally valid statement on forward or backward–forward bisimulations there is the corresponding universally valid statement on backward and forward–backward bisimulations. For that reason, our article deals only with forward and backward–forward bisimulations. In general, none of forward and backward bisimulations or backward–forward and forward–backward bisimulations have more practical applications than the other. For example, under the names right and left invariant equivalences, forward and backward bisimulation equivalences have been used by Ilie et al. [32–35] in reduction of the number of states of nondeterministic automata. It was shown that there are cases where one of them better reduces the number of states, but there are also other cases where the another one gives a better reduction. There are also cases where each of them individually causes a polynomial reduction of the number of states, but alternately using both types of equivalences the number of states can be reduced exponentially (cf. [33, Section 1]). It is also worth of mention that backward bisimulation equivalences were successfully applied in [55] in the conflict analysis of discrete event systems, while it was shown that forward bisimulation equivalences can not be used for this purpose.

As we already said, the main role of bisimulations is to model equivalence between the states of the same or different automata. However, bisimulations provide compatibility with the transitions, initial and terminal states of automata, but in general they do not behave like equivalences. A kind of relations which can be conceived as equivalences which relate elements of two possibly different sets appeared recently in [16] in the fuzzy framework. Here we consider the crisp version of these relations, the so-called uniform relations, where a *uniform relation* between two sets is defined as a complete and surjective relation φ satisfying the condition $\varphi \circ \varphi^{-1} \circ \varphi = \varphi$. The main aim of the paper is to show that the conjunction of two concepts, uniform relations and bisimulations, provides a very powerful tool in the study of equivalence between nondeterministic automata, where uniform relations serve as equivalence relations which relate states of two nondeterministic automata, and bisimulations ensure compatibility with the transitions, initial and terminal states of these automata. Our second goal is to employ the calculus of relations as a tool that will show oneself as very effective in the study of bisimulations. And third, we introduce and study a more general type of bisimulations, the so-called *weak bisimulations*. We show that equivalence of automata determined by weak bisimulations is closer to the language equivalence than equivalence determined by bisimulations, and we also show that they produce smaller automata than bisimulations when they are used in the the reduction of the number of states.

Our main results are the following. The main concepts and results from [16] concerning uniform fuzzy relations are translated to the case of ordinary relations, and besides, the proofs and some statements are simplified (cf. Theorems 3.1, 3.2 and 3.4). We also define the concept of the factor automaton with respect to an arbitrary equivalence, and prove two theorems that can be conceived as a version, for nondeterministic automata, of two well-known theorems of universal algebra: Second Isomorphism Theorem and Correspondence Theorem (cf. Theorems 4.1 and 4.2). Then we study the general properties of forward and backward-forward bisimulations. In cases where there is at least one forward or backward–forward bisimulation, we prove the existence of the greatest one, and we also show that the greatest forward bisimulation is a partial uniform relation (cf. Theorems 5.5 and 5.6). An algorithm that decides whether there is a forward bisimulation between nondeterministic automata was provided by Kozen in [39]. When there is a forward bisimulation, this algorithm also computes the greatest one. Here we give another version of this algorithm, and we also provide an analogous algorithm for backward–forward bisimulations (Theorems 6.3 and 6.5).

Given two automata $\mathcal{A}$ and $\mathcal{B}$ and a uniform relation $\varphi \subseteq A \times B$ between their sets of states, we show that φ is a forward bisimulation if and only if both its kernel E_A^φ and co-kernel E_B^φ are forward bisimulation equivalences on $\mathcal{A}$ and $\mathcal{B}$, and the function $\widetilde{\varphi}$ induced in a natural way by φ is an isomorphism between factor automata $\mathcal{A}/E_A^\varphi$ and $\mathcal{B}/E_B^\varphi$ (Theorem 7.2). Also, given two forward bisimulation equivalences E on $\mathcal{A}$ and F on $\mathcal{B}$, we show that there is a uniform forward bisimulation between $\mathcal{A}$ and $\mathcal{B}$ whose kernel and co-kernel are E and F if and only if the factor automata $\mathcal{A}/E$ and $\mathcal{B}/F$ are isomorphic (Theorem 7.3). Two automata $\mathcal{A}$ and $\mathcal{B}$ are defined to be FB-equivalent if there is a complete and surjective forward bisimulation