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Conditional diagnosability of balanced hypercubes under the PMC model

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ABSTRACT

Diagnosability is a critical metric for determining the reliability of a multiprocessor system. In 2005, a new measure for fault diagnosis of a system, namely conditional diagnosability, was proposed to improve the number of faulty processors identified. In this paper, we study the conditional diagnosability of balanced hypercubes under the PMC model and show that the conditional diagnosability of the n-dimensional balanced hypercube is $4n-3$ for $n \geqslant 1$.

1. Introduction

Due to continuous advances in technology, a multiprocessor system may consist of hundreds or thousands of processors (nodes). Some of these processors may be defective while the system is in operation. For the purpose of maintaining the reliability of a system, a process called fault diagnosis is initiated to identify faulty nodes in a multiprocessor system or a VLSI/WSI-oriented computing system. After a system has been diagnosed, identified faulty nodes are replaced by fault-free nodes. The diagnosability of a system is the maximum number of faulty nodes that are guaranteed to be identified in the system. The diagnosability of many multiprocessor systems has been investigated in the literature.

Several diagnosis models have been proposed for diagnosing faulty nodes in a multiprocessor system. One important model, namely the PMC model, was proposed by Preparata et al. [12]. In the PMC model, each node u is able to test each of its adjacent nodes v where u is called the tester and v is called the tested node. The outcome of a test performed by a fault-free tester is 1 (respectively, 0) if the tested node is faulty (respectively, fault-free); however, the outcome of a test performed by a faulty tester is unreliable. Under the PMC model, studies of diagnosis algorithms [18], diagnosability [2,4], strong diagnosability [5], and pessimistic diagnosability [9] can be found in the literature. A modification of the PMC model, called the BGM model [1], was introduced to provide a more realistic representation of systems whose nodes have a rather complex structure. Although the PMC and BGM models utilize the same testing strategy, the BGM model assumes that a faulty node is always tested as faulty, whether the tester is faulty or not. Another popular model, the comparison model [6,11,14], adopts a different testing strategy, in which a node (called a comparator) sends the same task to its two neighbors, and then compares their responses.

The hypercube network [13] is a well-known interconnection network. The balanced hypercube was proposed by Wu and Huang [15] to enhance some properties of the hypercube. Exactly like the hypercube, it is bipartite and vertex-transitive; however, the balanced hypercube is superior to the hypercube in that it has a smaller diameter than that of the hypercube

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and supports an efficient reconfiguration without changing the adjacent relationship among tasks [15]. Studies about desired properties of balanced hypercubes can be found in the literature [7,8,15,17].

Lai et al. [10] proposed a new measure of diagnosability, called conditional diagnosability. The conditional diagnosability of a system is the maximum number of faulty nodes guaranteed to be identified in the system under the condition that at least one of the neighbors of any node is not faulty. In [10], they also obtained the conditional diagnosability of the hypercubes under the PMC model. The conditional diagnosability of other networks has also been studied in the literature [16,19,20]. In this paper, we study the conditional diagnosability of balanced hypercubes under the PMC model. We show that the conditional diagnosability of the n-dimensional balanced hypercube, denoted by BH_n, is $4n - 3$ for $n \geqslant 1$.

The rest of this paper is organized as follows. Section 2 introduces some definitions and notations. In Section 3, we obtain the conditional diagnosability of the balanced hypercube under the PMC model. Finally, Section 4 concludes.

2. Preliminaries

An n-dimensional balanced hypercube BH_n [15] is defined as follows.

Definition 1. For $n \geqslant 1$, BH_n has 4^n vertices with addresses $(a_0,a_1,\ldots,a_{n-1})$ where a_i is a number with $a_i \in \{0,1,2,3\}$ for $0 \leqslant i \leqslant n-1$. Each vertex $(a_0,a_1,\ldots,a_{n-1})$ connects to the following $2n$ vertices:

$$(a_0 \pm 1, a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_{n-1}), \text{ and}$$
$$(a_0 \pm 1, a_1, \ldots, a_{i-1}, a_i + (-1)^{a_0}, a_{i+1}, \ldots, a_{n-1})$$

where i is an integer with $1 \leqslant i \leqslant n-1$.

BH_1 and BH_2 are illustrated in Fig. 1.

The balanced hypercube BH_n can be recursively defined.

Definition 2. BH_n is recursively constructed as follows:

(1) BH_1 is a cycle consisting of four vertices labeled as 0, 1, 2, 3 respectively.
(2) For $n \geqslant 2$, BH_n is consisted of four copies of BH_{n-1}, denoted by BH^i_{n-1}, for every integer i with $0 \leqslant i \leqslant 3$. Every vertex $(a_0,a_1,\ldots,a_{n-2},i)$ of BH^i_{n-1} connects to two extra vertices:
 (2.1) $(a_0 \pm 1,a_1,\ldots,a_{n-2},i+1)$ in BH^{i+1}_{n-1} if a_0 is even.
 (2.2) $(a_0 \pm 1,a_1,\ldots,a_{n-2},i-1)$ in BH^{i-1}_{n-1} if a_0 is odd.

The first element a_0 of vertex $(a_0,a_1,\ldots,a_{n-1})$ is called the *inner address*, and the other elements a_i, for all $1 \leqslant i \leqslant n-1$, are called *outer addresses*. Throughout this paper, we call a vertex with an odd inner address a black vertex and a vertex with an even inner address a white vertex, i.e. a black vertex is in $\{(a_0,a_1,\ldots,a_{n-1})|(a_0,a_1,\ldots,a_{n-1}) \in V(BH_n) \text{ and } a_0 \text{ is odd}\}$, and a white vertex is in $\{(a_0,a_1,\ldots,a_{n-1})|(a_0,a_1,\ldots,a_{n-1}) \in V(BH_n) \text{ and } a_0 \text{ is even}\}$. Hence, the sets of black and white vertices represent the two partite sets in BH_n. In what follows, we also write a vertex $(a_0,a_1,\ldots,a_{n-1})$ of BH_n as $a_0a_1\ldots a_{n-1}$ for brevity.

BH_n is a vertex transitive graph [15]. That is, for two arbitrary vertices x and y of BH_n, there is an automorphism T of BH_n such that $T(x) = y$.

A multiprocessor system can be modeled as a graph in which the vertices correspond to the processors and the edges correspond to the communication links between the processors. Let G be a simple graph. We denote by $V(G)$ and $E(G)$ the sets of vertices and edges of G respectively. Also, we denote by $|V(G)|$ and $|E(G)|$ the numbers of vertices and edges of G

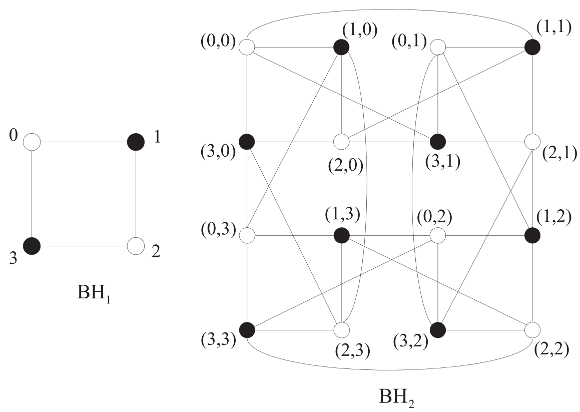

Fig. 1. Illustration of BH_1 and BH_2.