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New distance measures between intuitionistic fuzzy sets and interval-valued fuzzy sets

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ABSTRACT

Over the past several years, many studies have introduced distance and similarity measures between intuitionistic fuzzy sets (IFSs) and interval-valued fuzzy sets (IVFSs). This paper first reviews several widely used distance measures and then proposes two new distance measures. Moreover, the paper proves that the measures proposed satisfy the properties of the axiomatic definition for distance measures and introduces two corollaries. In addition, several numerical examples are provided to compare the second proposed distance measure with a number of existing distance measures. The results demonstrate that the second distance measure proposed can significantly overcome the drawback of information loss because it contains information regarding all of the points on the interval of a given IVFS. The paper concludes with suggestions for future research.

1. Introduction

Intuitionistic fuzzy sets (IFSs), originally introduced by Atanassov [1], represent one of the most influential generalizations of Zadeh's fuzzy set [48] and have been successfully used in various fields as a powerful tool for processing imprecise or vague information. Interval-valued fuzzy sets (IVFSs), proposed by Gorzalczany [14] and Turksen [32], represent another well-known extension of fuzzy set theory used to describe objective reality more realistically and precisely. These two theories are equipollent extensions of classical fuzzy set theory with different expressions [3] and were independently introduced to overcome some of the drawbacks of fuzzy set theory. Following the pioneering studies of Atanassov and Gorzalczany et al., many subsequent studies on the practical application of these theories have been performed in many fields, e.g., decision making [6,7,10,12,13,16,21,25,29,30,38–49,40–42], approximate reasoning [8,15,51], logic programming [4,5], pattern recognition [17,19,33], and cluster analysis [9,35,43,50].

In the aforementioned practical applications, the similarity and distance measures between IFSs and IVFSs are of high importance. The similarity and distance of IFSs are counterparts representing two aspects of the same measure. The purpose of the similarity measure is to estimate the degree of similarity between IFSs. In contrast, the distance describes the discrepancy between IFSs. Many researchers have conducted extensive studies on similarity and distance measures between IFSs. Atanassov [2] and Szmidt and Kacprzyk [26,27] proposed several methods for calculating the distance measure between IFSs using the Hamming distance and the Euclidean distance. An entropy method [33] was also developed to measure the distance between IFSs. Liang and Shi [23] and Mitchell [24] improved the similarity measures proposed by Li and Cheng [20]. Wang and Xin [34] also proposed methods for calculating the distance measures of IFSs and applied these to pattern recognition. Hung and Yang [18] proposed two new similarity measures between IFSs and applied them to evaluate students'

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answerscripts. Xu and Chen [44] provided a comprehensive overview of the distance and similarity measures of IFSs and proposed additional continuous distance and similarity measures for IFSs. Based on the method of Szmidt and Kacprzyk [31], Xu and Yager [45] proposed an improved degree of similarity between IFSs. Xia and Xu [46] introduced a series of similarity measures for intuitionistic fuzzy values based on intuitionistic fuzzy operators. Wei et al. [36] developed an approach for constructing similarity measures using entropy measures for IVFSs. Ye [47] presented a cosine similarity measure of IFSs. Hwang et al. [19] proposed a similarity measure of IFSs based on the Sugeno integral. Li et al. [22] discussed the relationship between similarity measures and the entropy of IFSs and defined new similarity measures based on the entropy of IFSs. In the abovementioned measures, the degree of similarity and distance are obtained based mainly on particular points, e.g., midpoints or endpoints on the intervals of IVFSs, which inevitably leads to information loss and even ineffectiveness of the distance measures in some cases. Therefore, we propose new distance measures and compare them with some existing distance measures.

The rest of this paper is organized as follows. In Section 2, a brief introduction to the basic notions of IFSs and IVFSs is provided. Section 3 reviews widely used distance and similarity measures between IFSs and IVFSs. In Section 4, two new distance measures are introduced. In Section 5, several numerical examples are provided to compare the proposed distance measure with existing distance measures. Section 6 discusses our conclusions and provides suggestions for future work.

2. Preliminaries

An IFS A in X was described by Atanassov [1] as

$$A = \{(x, \mu_A(x), \upsilon_A(x)) | x \in X\},$$

where the functions $\mu_A(x)$: $X \to [0,1]$ and $\upsilon_A(x)$: $X \to [0,1]$ represent the degree of membership and non-membership of the element x in A, respectively. For any $x \in X$, the following condition holds: $0 \leqslant \mu_A(x) + \upsilon_A(x) \leqslant 1$.

$\pi_A(x)$ is called the intuitionistic index or degree of indeterminacy of x to A: $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$. Clearly, if $\pi_A(x) = 0$, the IFS A is reduced to a fuzzy set.

Let IVFS $B = [\mu_{BL}(x), \mu_{BU}(x)]$, where$0 \leqslant \mu_{BL}(x) \leqslant \mu_{BU}(x) \leqslant 1$. Atanassov and Gargov [3] showed that IFSs and IVFSs are equipollent extensions of classical fuzzy sets that use mapping relationships: the map f assigns an IFS $A = f(B)$ described by.

$\mu_A(x) = \mu_{BL}(x)$ and $\upsilon_A(x) = 1 - \mu_{BU}(x)$ to every IVFS B.

In addition, the map g assigns an IVFS $B = g(A)$ given by.

$\mu_{BL}(x) = \mu_A(x)$ and $\mu_{BU}(x) = 1 - \upsilon_A(x)$ to every IFS A.

Thus, all of the definitions, theorems, and corollaries introduced in this manuscript for IFSs can be adapted to the analysis of IVFSs.

A few basic definitions of IFSs are presented below to facilitate further discussion.

Definition 2.1. [3] If A and B are two IFSs of set X, then

(1) $A \subseteq B$ if and only if $\forall x \in X$, $\mu_A(x) \leqslant \mu_B(x)$, and $\upsilon_A(x) \geqslant \upsilon_B(x)$;
(2) $A = B$ if and only if $\forall x \in X$, $\mu_A(x) = \mu_B(x)$, and $\upsilon_A(x) = \upsilon_B(x)$;
(3) $A^C = \{(x, \upsilon_A(x), \mu_A(x)) | x \in X\}$, where A^C is the complement of A.

Li and Cheng [20] and Mitchell [24] introduced the following definition of similarity between IFSs.

Definition 2.2. A mapping S: IFSs $(X) \times$ IFSs $(X) \to [0,1]$. $S(A,B)$ is said to be the degree of similarity between $A \in$ IFSs (X) and $B \in$ IFSs (X) if $S(A,B)$ satisfies the following properties:

(P1) $0 \leqslant S(A,B) \leqslant 1$;
(P2) $S(A,B) = 1$ if and only if $A = B$;
(P3) $S(A,B) = S(B,A)$; and
(P4) $S(A,C) \leqslant S(A,B)$ and $S(A,C) \leqslant S(B,C)$ if $A \subseteq B \subseteq C$, where $C \in$ IFSs (X).

Wang and Xin [34] introduced the axiomatic definition of distance.

Definition 2.3. Let d be a mapping d: IFSs $(X) \times$ IFSs $(X) \to [0,1]$. If $d(A,B)$ satisfies the following properties, $d(A,B)$ is a distance measure between IFSs A and B.

(DP1) $0 \leqslant d(A,B) \leqslant 1$;
(DP2) $d(A,B)=0$ if and only if $A = B$;
(DP3) $d(A,B) = d(B,A)$; and
(DP4) $d(A,C) \geqslant d(A,B)$ and $d(A,C) \geqslant d(B,C)$ if $A \subseteq B \subseteq C$, where $C \in$ IFSs (X).