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Reasoning with the finitely many-valued Łukasiewicz fuzzy Description Logic $\mathcal{SROIQ}$ ☆

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ABSTRACT

Fuzzy Description Logics are a formalism for the representation of structured knowledge affected by imprecision or vagueness. They have become popular as a language for fuzzy ontology representation. To date, most of the work in this direction has focused on the so-called Zadeh family of fuzzy operators (or fuzzy logic), which has several limitations. In this paper, we generalize existing proposals and show how to reason with a fuzzy extension of the logic $\mathcal{SROIQ}$, the logic behind the language OWL 2, under finitely many-valued Łukasiewicz fuzzy logic. We show for the first time that it is decidable over a finite set of truth values by presenting a reasoning preserving procedure to obtain a non-fuzzy representation for the logic. This reduction makes it possible to reuse current representation languages as well as currently available reasoners for ontologies.

1. Introduction

In the last years, the use of ontologies as formalisms for knowledge representation in many different application domains has grown significantly. Ontologies have been successfully used as part of expert and multiagent systems, as well as a core element in the Semantic Web, which proposes to extend the current web to give information a well-defined meaning [3].

An ontology is defined as an explicit and formal specification of a shared conceptualization [24], which means that ontologies represent the concepts and the relationships in a domain promoting interrelation with other models and automatic processing. Ontologies allow adding semantics to data, making knowledge maintenance, information integration, and reuse of components easier.

The current standard language for ontology creation is the Web Ontology Language (OWL [63]), which comprises three sublanguages of increasing expressive power: OWL Lite, OWL DL, and OWL Full. OWL Full is the most expressive level, but reasoning within it becomes undecidable; OWL Lite has the lowest complexity; and OWL DL is a balanced tradeoff between expressiveness and reasoning complexity. However, since its first development, several limitations on expressiveness of OWL have been identified, and consequently several extensions to the language have been proposed [55]. Among them, the most significant is OWL 2 [18], its most likely immediate successor which is currently a Proposed Recommendation at W3C [64].

Description Logics (DLs for short) [1] are a family of logics for representing structured knowledge. Each logic is denoted by using a string of capital letters which identify the constructors of the logic and therefore its complexity. DLs have proved to

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be very useful as ontology languages [2]. For instance, OWL Lite, OWL DL and OWL 2 are close equivalents to $\mathcal{SHIF}(\mathbf{D})$, $\mathcal{SHOIN}(\mathbf{D})$ and $\mathcal{SROIQ}(\mathbf{D})$, respectively [31].

Nevertheless, it has been widely pointed out that classical ontologies are not appropriate to deal with imprecise and vague knowledge, which is inherent to several real world domains [49].

Fuzzy logic is a suitable formalism to handle these types of knowledge. In the setting of fuzzy logics, the convention prescribing that a statement is either true or false is changed. A more refined range is used, in such a way that every fuzzy statement has a degree of truth $\alpha \in [0,1]$ [28].

Several fuzzy extensions of DLs can be found in the literature (see [41] for a survey), as the theoretical basis of fuzzy ontologies. Fuzzy ontologies have proved to be useful in several applications, such as Chinese news summarization [39], semantic help-desk support [46], ontology-based query enrichment [37], information retrieval [15], or image interpretation, (e.g. recognition of brain structures in 3D magnetic resonance images [32]). There are also a lot of applications in the Semantic Web field (see for example [47,17]) and, more generally, in the Internet [49].

In fuzzy logic, all classical set operations are extended to the fuzzy case. The intersection, union, complement and implication set operations are performed by a t-norm function $\otimes$, a t-conorm function $\oplus$, a negation function $\ominus$, and an implication function $\Rightarrow$, respectively. These functions or fuzzy operators are grouped in families, also simply called fuzzy logics.

It is well known that different families of fuzzy operators lead to fuzzy DLs with different properties. There are three main fuzzy logics: Łukasiewicz, Gödel and Product. It is also common to consider the fuzzy set operators originally proposed by Zadeh: Gödel conjunction and disjunction, Łukasiewicz negation and Kleene–Dienes implication.

Although there has been a relatively significant amount of work in extending DLs with fuzzy set theory [41], most of the existing works restrict themselves to Zadeh fuzzy logic (see Section 2.4 for a definition and Section 5 for a detailed summary of the state of the art in fuzzy DLs).

This paper provides a reasoning algorithm for Łukasiewicz fuzzy $\mathcal{SROIQ}$ over a finite set of truth values, the logic behind OWL 2. This is the first reasoning algorithm for such an expressive logic, for which decidability was not known.

Compared to Zadeh logic, our proposal provides several advantages:

- Łukasiewicz fuzzy logic is more general than Zadeh fuzzy logic.
- The implication of Zadeh fuzzy logic (Kleene–Dienes implication) has some counter-intuitive effects [27,4]. For instance, a concept does not fully subsume itself. Łukasiewicz implication solves these problems.
- The t-norm of Zadeh and Gödel fuzzy logics (the minimum t-norm) is idempotent and hence it is not Pareto optimal [48]. This is problematic in some applications such as fuzzy matchmaking [48].

Defining a fuzzy DL brings about that standard languages would no longer be appropriate, new fuzzy languages should be used and hence the large number of resources available should be adapted to the new framework, requiring an important effort. An alternative is to represent fuzzy DLs using non-fuzzy DLs and to reason using these representations. Our reasoning algorithm will provide such a non-fuzzy representation.

The remainder of this work is organized as follows. Section 2 overviews some necessary background. Section 3 describes a fuzzy extension of $\mathcal{SROIQ}$ and particularizes it to the case of Łukasiewicz fuzzy logic. Section 4 depicts a reduction into $\mathcal{SROIQ}$. Section 5 reviews some related work. Finally, Section 6 sets out some conclusions and ideas for future work.

2. Preliminaries

This section provides some basic background. Section 2.1 quickly overviews $\mathcal{SROIQ}$ [30], the DL which will be mainly treated throughout this paper. Then, Section 2.4 refreshes some basic ideas in mathematical fuzzy logic [28].

2.1. The Description Logic $\mathcal{SROIQ}$

$\mathcal{SROIQ}$ extends $\mathcal{ALC}$ standard DL [52] with transitive roles ($\mathcal{ALC}$ plus transitive roles is called $\mathcal{S}$), complex role axioms ($\mathcal{R}$), nominals ($\mathcal{O}$), inverse roles ($\mathcal{I}$) and qualified number restrictions ($\mathcal{Q}$).

2.2. Syntax

$\mathcal{SROIQ}$ assumes three alphabets of symbols, for concepts, roles and individuals. In DLs, complex concepts and roles can be built using different concept and role constructors. In $\mathcal{SROIQ}$, the concepts (denoted C or D) and roles (R) can be built inductively from atomic concepts (A), atomic roles (R_A), top concept $\top$, bottom concept $\bot$, named individuals (o_i), simple roles (S, which will be defined below) and universal role U, as shown in Table 1, where n, m are natural numbers ($n \geqslant 0,\ m > 0$), $x, y \in \Delta^{\mathcal{I}}$ are abstract individuals and $\sharp X$ denotes the cardinality of the set X.

Example 2.1. `Man` and `Woman` are atomic concepts. `hasChild` and `likes` are atomic roles. $\texttt{Man} \sqcap \geqslant 2\texttt{hasChild}.\texttt{Woman}$ is a complex concept representing a father with at least two daughters. $\exists\texttt{likes}.\texttt{Self}$ represents a narcisist.