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Mean-square data-based controller for nonlinear polynomial systems with multiplicative noise

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ABSTRACT

This paper presents the mean-square optimal data-based quadratic-Gaussian controller for stochastic nonlinear polynomial systems with a polynomial multiplicative noise, a linear control input, and a quadratic criterion over linear observations. The mean-square optimal closed-form controller equations are obtained using the separation principle, whose applicability to the considered problem is substantiated. As an intermediate result, the paper gives a closed-form solution of the optimal regulator (control) problem for stochastic nonlinear polynomial systems with a polynomial multiplicative noise, a linear control input, and a quadratic criterion. Performance of the obtained mean-square optimal data-based controller is verified in the illustrative example against the conventional LQG controller that is optimal for linearized systems. Simulation graphs demonstrating overall performance and computational accuracy of the designed optimal controller are included.

1. Introduction

Although the mean-square optimal LQG controller problem for linear systems was solved in 1960s, based on the solutions to the optimal filtering [15] and optimal regulator [9,17] problems, the optimal controller for nonlinear systems has to be determined using the nonlinear filtering theory (see [14,16,18]) and the general principles of maximum [9] or dynamic programming [7], which do not provide an explicit form for the optimal control in most cases. However, taking into account that the optimal filtering and control problems can be explicitly solved in a closed-form in the linear case, and the optimal controller can be then obtained using the separation principle [9,17], this paper exploits the same approach for designing the optimal controller for polynomial systems with linear control input over linear observations. The designed optimal solution is based on the recently obtained optimal filter and regulator for polynomial systems states. Thus, this paper continues a long tradition of the optimal control design for nonlinear systems (see, for example, [1,11,12,19,22,29,30,32,37]) and not so long research on the optimal closed-form filter design for nonlinear [8,13,23–25,33,35], and in particular, polynomial [3,4] systems. Nevertheless, to the best of authors' knowledge, the optimal closed-form controller design for polynomial systems with polynomial multiplicative noises has not been yet considered in the literature, due to the absence of closed-form solutions to the optimal filtering and control problems for that class of systems.

This paper presents solution to the mean-square optimal data-based quadratic-Gaussian controller problem for stochastic nonlinear polynomial systems with a polynomial multiplicative noise [2,31], a linear control input, and a quadratic criterion over linear observations. First, the separation principle is substantiated for polynomial systems with a polynomial

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multiplicative noise, a linear control input, and a quadratic criterion over linear observations. Then, the paper gives a closed-form solution to the optimal regulator (control) problem for stochastic nonlinear polynomial systems with a polynomial multiplicative noise, a linear control input, and a quadratic criterion. The obtained solution consists of a linear feedback control law and two differential equations, linear and Riccati ones, for forming the optimal control gain matrix. This result is proven in Appendix A. Finally, based on that closed-form optimal control problem solution, the mean-square optimal filter for stochastic polynomial systems with a polynomial multiplicative noise over linear observations [4], and the separation principle, the paper presents the optimal solution to the original quadratic-Gaussian controller problem, which has essentially the same structure as the solved optimal regulator (control) problem plus the variance equation for forming the optimal filter gain matrix. All four differential equations included in the optimal controller are interconnected. Note that the obtained controller is designed using the stochastic data-based observation process, which enables one to properly formalize and process on-line distinctly organized data incoming from telemetry, networks, Poisson flows, etc. The previous engineering applications of similar controller algorithms can be found in [6,10,26–28,34,36,38].

Finally, performance of the designed optimal controller for stochastic nonlinear polynomial systems with a polynomial multiplicative noise, a linear control input, and a quadratic criterion over linear observations is verified in the illustrative example against the conventional LQG controller that is optimal for a linearized system.

The paper is organized as follows. In Section 2, the mean-square optimal data-based controller problem is stated and solved for stochastic nonlinear polynomial systems with a polynomial multiplicative noise, a linear control input, and a quadratic criterion over linear observations. First, the separation principle is substantiated for the considered class of polynomial systems. Next, a closed-form solution of the optimal regulator (control) problem is designed for nonlinear polynomial systems with a polynomial multiplicative noise, a linear control input, and a quadratic criterion. This result is proven in Appendix A. Finally, the mean-square optimal solution to the original linear-quadratic controller problem is given. Section 3 presents an example illustrating the efficiency of the designed mean-square optimal controller for nonlinear polynomial systems against the conventional LQG controller. Simulation graphs verifying overall performance and computational accuracy of the designed optimal controller are included.

In this paper, a controller problem is referred to as the control problem for a system with unmeasured state values, where the control law is based on an estimate obtained as a result of application of a state filter. The control (regulator) problem is referred to as the control problem for a system with available state values, where the control law is based on the system states themselves.

2. Optimal controller problem

2.1. Problem statement

Let (Ω, F, P) be a complete probability space with an increasing right-continuous family of σ-algebras F_t, $t \geqslant t_0$, and let $(W_1(t), F_t, t \geqslant t_0)$ and $(W_2(t), F_t, t \geqslant t_0)$ be independent Wiener processes. The F_t-measurable random process $(x(t), y(t))$ is described by a nonlinear differential equation with a polynomial drift term for the system state with polynomial multiplicative noise

$$\begin{aligned} dx(t) &= f(x,t)dt + B(t)u(t)\,dt + b(x,t)\,dW_1(t), \\ x(t_0) &= x_0, \end{aligned} \tag{1}$$

and a linear differential equation for the observation process

$$dy(t) = (A_0(t) + A(t)x(t))\,dt + G(t)\,dW_2(t). \tag{2}$$

Here, $x(t) \in R^n$ is the state vector, $u(t) \in R^l$ is the control input, and $y(t) \in R^m$ is the linear observation vector, $m \leqslant n$. The initial condition $x_0 \in R^n$ is a Gaussian vector such that $x_0, W_1(t) \in R^p$, and $W_2(t) \in R^q$ are independent. The observation matrix $A(t) \in R^{m\times n}$ is not supposed to be invertible or even square. It is assumed that $G(t)G^T(t)$ is a positive definite matrix, therefore, $m \leqslant q$. All coefficients in (1) and (2) are deterministic functions of appropriate dimensions.

The nonlinear functions $f(x,t)$ and $b(x,t)$ are considered polynomial of n variables, components of the state vector $x(t) \in R^n$, with time-dependent coefficients. Since $x(t) \in R^n$ is a vector, this requires a special definition of the polynomial for $n > 1$. In accordance with [6], a p-degree polynomial of a vector $x(t) \in R^n$ is regarded as a p-linear form of n components of $x(t)$

$$f(x,t) = a_0(t) + a_1(t)x + a_2(t)xx^T + \cdots + a_p(t)x \cdots_{p\ times} \cdots x, \tag{3}$$

where a_0 is a vector of dimension n, a_1 is a matrix of dimension $n \times n$, a_2 is a 3D tensor of dimension $n \times n \times n$, a_p is an $(p+1)$D tensor of dimension $n \times \cdots_{(p+1) times} \cdots \times n$, and $x \times \cdots_{p\ times} \cdots \times x$ is a pD tensor of dimension $n \times \cdots_{p\ times} \cdots \times n$ obtained by p times spatial multiplication of the vector $x(t)$ by itself. Such a polynomial can also be expressed in the summation form

$$f_k(x,t) = a_{0k}(t) + \sum_i a_{1\ ki}(t)x_i(t) + \sum_{ij} a_{2\ kij}(t)x_i(t)x_j(t) + \cdots + \sum_{i_1 \ldots i_p} a_{p\ ki_1\ldots i_p}(t)x_{i_1}(t)\ldots x_{i_p}(t), \quad k, i, j, i_1 \ldots\ i_p = 1, \ldots, n.$$