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Entropy, similarity measure and distance measure of vague soft sets and their relations

Chang Wang *, Anjing Qu

Department of Mathematics, Northwest University, Xi'an, Shaanxi 710127, PR China

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ABSTRACT

Soft set theory offers a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. A vague soft set is a combination of a vague set and a soft set. In this paper, we introduce axiomatic definitions of entropy, similarity measure and distance measure for vague soft sets, and some formulas have also been put forward to calculate them. Furthermore, 13 theorems are proposed showing how the entropy, the similarity measure and the distance measure for vague soft sets can be deduced from each other. Some formulas have also been put forward to calculate the entropy, the similarity measure and the distance measure of vague soft sets by these relations.

1. Introduction

Researchers in economics, engineering, environment science, the social science, medical science, business, management, and many other fields deal daily with the complexities of modeling uncertain data. Classical methods are not always successful, because the uncertainties appearing in these domains may be of various types. Probability theory, fuzzy set theory [47], intuitionistic fuzzy set theory [2], vague set theory [16], interval mathematics [3,17], and other mathematical tools are well-know and often useful approaches to describing uncertainty. However, all of these theories have their own difficulties which have been pointed out in [36]. Molodtsov suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tools of these theories. To overcome these difficulties, Molodtsov [36] introduced the concept of soft sets as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Since then, many researches have investigated soft sets and have established some significant conclusions. For example, Jun and Park [25] proposed the notion of soft ideals and idealistic soft BCK/BCI-algebras, and constructed several examples. Ali et al. [1] corrected some errors in earlier studies and proposed some new operations on soft sets. Çağman and Enginoğlu [7] redefined the operations of soft sets and constructed a uni-int decision making method which selected a set of optimum elements from the alternatives, they also defined soft matrices, a matrix representation of soft sets, and constructed a soft max-min decision making method [8]. Jiang et al. [24] proposed a novel approach to semantic decision making by using ontology-based soft sets and ontology reasoning. Qin and Hong [37] introduced the concept of soft equality and some related properties were derived, some equivalent conditions for soft sets being equal were also given. Herawan and Deris [19] presented an alternative approach for mining regular association rules and maximal association rules from transactional datasets using soft set theory.

* Corresponding author. Tel.: +86 29 88308430.
E-mail address: cwang@nwu.edu.cn (C. Wang).

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It is worth noting that all of above works are built on the classical soft set theory. The generalizations of soft sets to environments in which uncertainty is a factor have become a rapidly progressing research area receiving much attention in recent years. Maji et al. [31] first introduced the concept of fuzzy soft sets by combining fuzzy sets and soft sets. Majumdar and Samanta [34] further generalized the concept of fuzzy soft sets and some of their properties were studied, and relations on generalized fuzzy soft sets were also discussed by them. Yang et al. [44] introduced the concept of interval-valued fuzzy soft set, which is a combination of interval-valued fuzzy sets and soft sets. Xiao et al. [41] introduced the notion of exclusive disjunctive soft sets and gave an application of these new sets. Maji et al. [32,33] initiated the notion of intuitionistic fuzzy soft sets by integrating the intuitionistic fuzzy sets with soft sets. By combing the vague set and the soft set, Xu et al. [42] introduced the notion of vague soft sets, derived its basic properties and illustrated its potential applications. Jiang et al. [22] constructed a new soft set model called interval-valued intuitionistic fuzzy soft sets by integrating the interval-valued intuitionistic fuzzy sets and soft sets.

Entropy, similarity measure and distance measure are three very important notions for measuring uncertain information. The entropy of a fuzzy set describes the fuzziness degree of a fuzzy set, and was first introduced by Zadeh [47] in 1965. Although the name entropy was chosen due to the concept's intrinsic similarity to Shannon entropy, the two functions measure fundamentally different types of uncertainty [12,39]. Basically, the Shannon entropy measures the uncertainty in a random experiment and is derived from the probability theory. In 1972, De Luca and Termini [13] suggested the axiomatic construction of non-probabilistic entropy of fuzzy sets and referred to Shannon's probability entropy, interpreting it as a measure of the amount of information. Later, Kaufmann [26] proposed to measure the degree of fuzziness of a fuzzy set by a metric distance between its membership function and the membership function of its nearest crisp set. Another method presented by Yager [43] was to view the fuzziness degree of a fuzzy set in terms of a lack of distinction between the fuzzy set and its complement. Bustince and Burillo [5] introduced the concept of entropy of intuitionistic fuzzy set and the interval-valued fuzzy set in 1996. Szmidt and Kacprzyk [39] extended De Luca and Termini's axioms and proposed an entropy measure for intuitionistic fuzzy set. Zeng and Li [48] expressed the axioms of Szmidt and Kacprzyk using the notation of interval-valued fuzzy set. Farhadinia [15] generalized some results on the entropy of interval-valued fuzzy sets based on the intuitionistic distance and its relationship with similarity measure.

The similarity measure and distance measure both indicate the degree of similarity of two sets, and the notions have also attracted many scholars. For example, Zeng et al. [49] investigated the relationship among the inclusion measure, the similarity measure, and the fuzziness of fuzzy sets. Chen [9] first put forward the concept of similarity measure for vague sets and gave a computation formula. From [29], we know that many scholars presented formulas to calculate the similarity measure of vague sets from different viewpoints. Szmidt and Kacprzyk [38] proposed four distance measures between vague sets, which were in some extent based on the geometric interpretation of intuitionistic fuzzy sets, and have some good geometric properties. Wang and Xin [40] provided a more generalized definition of distance measures between intuitionistic fuzzy sets, and then proposed several new distance measures. Li et al. [28] investigated the relationship between the similarity measure and the entropy of intuitionistic fuzzy sets. Hwang et al. [21] proposed a new similarity measure formula for intuitionistic fuzzy sets induced by the Sugeno integral. Furthermore, the applications of similarity measure and distance measure can be found in [4,14,27,35,46].

By definition, a soft set is a parameterized family of subsets of the universal set. In other words, a soft set is a mapping from a set of parameters to the power set of an initial universe set. In the real world, the difficulty is that the objects in the universal set may not precisely satisfy the problem's parameters, which usually represent some attributes, characteristics, or properties of the objects in the universal set. The concept of fuzzy soft sets proposed in [31] partially resolves this difficulty, but falls short in dealing with additional complexity – that is, the mapping may be too vague. It is, therefore, desirable to extend soft set theory and fuzzy soft set theory using the concept of vague set theory. Vague set theory is actually an extension of fuzzy set theory and vague sets are regards as a special case of context-dependent fuzzy sets. The basic concepts of vague set theory and its extensions, as well as some interesting applications can be found in [10,11,20,45,51]. Vague soft set theory makes descriptions of the object world more realistic, practical and accurate, at least in some cases, making it a very promising tool. Since vague sets are equivalent to intuitionistic fuzzy sets [6], so vague soft sets are equivalent to intuitionistic fuzzy soft sets. Some scholars have studied intuitionistic fuzzy soft sets from different aspects. For example, Gunduz and Bayramov [18] introduced the concept of an intuitionistic fuzzy soft module and some operations on intuitionistic fuzzy soft sets were given, they also studied some of its basic properties. Jiang et al. [22] proposed the notion of the interval-valued intuitionisitc fuzzy soft set, the complement, and, or, union, intersection, necessity and possibility operations were defined on interval-valued intuitionistic fuzzy soft sets, and the basic properties of interval-valued intuitionistic fuzzy soft sets were discussed. They [23] also presented an adjustable approach to intuitionistic fuzzy soft sets based decision making by using level soft sets of intuitionistic fuzzy soft sets and gave some illustrative examples, the weighted intuitionistic fuzzy soft sets were introduced and its application to decision making was investigated. Zhang [52] proposed a novel approach to intuitionistic fuzzy soft set based decision making problems using rough set theory. However, there has been rather little work completed for entropy, similarity measure and distance measure in the context of intuitionistic fuzzy soft sets and their relations. The purpose of this paper is to further extend the concept of vague soft set theory proposed by Xu et al. in [42]. In this paper, we will present the axiomatic definitions of entropy, similarity measure and distance measure of vague soft sets and study the basic relations between them.

The rest of this paper is organized as follows. Section 2 recalls some basic concepts of vague sets, soft sets and vague soft sets. In Section 3, we introduce the definitions of entropy, similarity measure and distance measure of vague soft sets, and