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Fuzzy logic-based generalized decision theory with imperfect information

Rafik Aliev [a,*], Witold Pedrycz [b,c], Bijan Fazlollahi [d], O.H. Huseynov [a], A.V. Alizadeh [e], B.G. Guirimov [e]

[a] Department of Computer-Aided Control Systems, Azerbaijan State Oil Academy, Baku, Azerbaijan
[b] Department of Electrical & Computer Engineering, University of Alberta, Edmonton, Canada AB T6R 2G7
[c] System Research Institute, Polish Academy of Sciences, Warsaw, Poland
[d] Center for Business Development in Transitional Economies, Georgia State University, Atlanta, GA 30302-3989, USA
[e] Azerbaijan Association of "Zadeh's Legacy", Baku, Azerbaijan

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ABSTRACT

The existing decision models have been successfully applied to solving many decision problems in management, business, economics and other fields, but nowadays arises a need to develop more realistic decision models. The main drawback of the existing utility theories starting from von Neumann–Moregnstern expected utility to the advanced non-expected models is that they are designed for laboratory examples with simple, well-defined gambles which do not adequately enough reflect real decision situations. In real-life decision making problems preferences are vague and decision-relevant information is imperfect as described in natural language (NL). Vagueness of preferences and imperfect decision relevant information require using suitable utility model which would be fundamentally different to the existing precise utility models. Precise utility models cannot reflect vagueness of preferences, vagueness of objective conditions and outcomes, imprecise beliefs, etc. The time has come for a new generation of decision theories. In this study, we propose a decision theory, which is capable to deal with vague preferences and imperfect information. The theory discussed here is based on a fuzzy-valued non-expected utility model representing linguistic preference relations and imprecise beliefs.

"It is better to be roughly right than precisely wrong"
John Maynard Keynes

1. Introduction

Decision making theory is a holy grail of numerous studies in management science, economics and other areas. It comprises a broad diversity of approaches to modeling behavior of a decision maker realized under various information frameworks. In essence, the solution to the decision making problem is defined by a preferences framework and a type of decision-relevant information. In its turn preference and decision-relevant information frameworks are closely related. One of the approaches to formally describe preferences on the base of decision-relevant information is the use of a utility

* Corresponding author. Tel.: +994 125984509.
E-mail addresses: raliev@asoa.edu.az (Rafik Aliev), pedrycz@ee.ualberta.ca (W. Pedrycz), bijan@gsu.edu (B. Fazlollahi), oleg_huseynov@yahoo.com (O.H. Huseynov), akifoder@yahoo.com (A.V. Alizadeh), b.guirimov@azbtc.com (B.G. Guirimov).

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function. Utility function is a quantitative representation of preferences of a decision-maker (DM) and any scientifically grounded utility model comes with the underlying preference assumptions.

The existing utility models are developed within the two main directions of decision making theory: decision making under ignorance, decision making under risk and decision making under ambiguity.

Decision making under ignorance [56,69] is characterized by an absence of any information about probabilities of events. Unfortunately, this is an ideal view on decision-relevant information, because in real-life a DM almost always has some limited information about probabilities. The decision making methods developed for situations of ignorance include Laplace insufficient reason criterion, Savage minimax regret criterion, Hurwitz criterion, Wald maximin solution rule, etc. Maximin solution rule models extreme pessimism in decision making, whereas its generalization, Hurwitz criterion uses linear combination of pessimistic and optimistic solutions.

In decision making under risk [27,100,102,110,111,118] it is supposed that precise objective or subjective probabilities of states of nature and precise outcomes are available. Subjective probabilities [102] are used when objective probabilities are unknown. The main methods of decision making under risk are von Neumann and Morgenstern expected utility (EU) [118], subjective expected utility (SEU) [102] and Kahneman and Tversky Prospect theory (PT) [47,111]. As it was shown by many experiments, the use of precise objective or subjective probabilities appeared non-realistic [15,25,40,60,117]. On the other hand, even if objective probabilities were known, beliefs of a DM do not coincide with them but are affected by some kind of distortion – they are transformed into so-called decision weights [47,90,111].

A large number of studies is devoted to decision making under ambiguity [25,45,28,53,54,49,38,19,65,73,43,52,60,77]. Ambiguity is commonly referred to as uncertainty with respect to probabilities – the cases when probabilities are not known or are supposed to vary within some range. The terms 'uncertainty' and 'ambiguity' are not always clearly distinguished and defined but, in general, are related to non-probabilistic uncertainty. In turn, decision making under uncertainty often is considered as an extreme non-probabilistic case – when no information on probabilities is available. From the other side, this case is also termed as decision making under complete ignorance.At the same time, sometimes, this is considered as ambiguity represented by simultaneous consideration of all the probability distributions. The studies on decision making under ambiguity are conducted in two directions – a development of models based on multiple probability distributions, called multiple priors models [58,59,42], and a formation of approaches based on non-additive measures [120–123]. Mainly, these models consider so-called ambiguity aversion as a property of human behavior to generally prefer outcomes related to non-ambiguous events to those related to ambiguous ones.

The well-known approach developed for multiple priors is Maximin EU (MMEU) and its development [25,28,52–55,26,100,78,43–46,61,80]. In this criterion, an alternative is evaluated by minimal or maximal expected utility with respect to all possible probability distributions. In [52] they suggest to use convex combination of minimal and maximal expected utilities.

In general, multiple priors are much more adequate but still a poor formulation of probability-relevant information available for a DM – in real-world problems a DM usually has some information that allows determining which priors are more and which are less relevant. For addressing this issue, models with second-order probabilities were suggested [19,29,105,79]. In [79] they suggest so-called 'smooth ambiguity' model which generalizes the existing MMEU models. In this model a subjective probability measure reflects DM's belief on whether a considered subset of multiple priors contains a 'true' prior. The use of these models is a step toward forming a more adequate information structure. However, a construction of a second-order probability distribution over first-order probabilities become doubtful as the latter cannot be known precisely [25]. Second-order precise probability model is a non-realistic description of human beliefs characterized by imprecision and associated with some psychological aspects that need to be considered as well. The other disadvantage of the belief representation suggested in [79] is that the problem of investigation of consistency of subjective probability-relevant information is not discussed – consistent multiple priors are supposed to be given in advance. However, a verification of consistency of beliefs becomes a very important problem. An extensive investigation of this issue is covered in [122].

The alternative approach to model imperfect information about probabilities is the use of imprecise probabilities, cf. [18,50,75,76,39,114,115,83,122,98,67,91,72,116]. Some intuitively acceptable and useful interpretations of imprecise probabilities are interval probabilities [39,67], fuzzy (linguistic) probabilities [48,127,101], to name a few viable alternatives. The first fundamental study in this framework was the Walley's theory of imprecise probabilities [122]. The key concept of the theory is the lower prevision, which can be used to model evaluations like lower and upper probabilities, belief functions, additive probability measures, etc. and does not impose any assumptions on the type of probability distributions. However, this theory often requires solving very complicated optimization problems.

In [67] they suggest an approach for decisions based on interval probabilities where the latter are obtained on the base of pairwise comparison of likelihood of events.

The important class of approaches to problems when a DM is uncertain about probabilities deals with imprecise hierarchical models [32,48,62,88,107,113,139]. In these models imprecise probabilities of states of nature are assigned at the first (lower) level. The second level is used to represent imprecise probability describing a DM's or experts' confidence about imprecise probability being assigned at the first level. However, most of the works devoted to hierarchical models deals with a large number of optimization problems.

One of the main models in this realm of application of non-additive measures (fuzzy measures) is Choquet expected utility (CEU) based on the Choquet integral [30]. As mentioned in [126], fuzzy measure is a unified description of various types of characterizations of uncertainty such as randomness, lack of specificity, and imprecision [119,35,34,31,37,22,108,74]. CEU is