Information Sciences 187 (2012) 179–203

Contents lists available at SciVerse ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

A type-2 fuzzy c-regression clustering algorithm for Takagi–Sugeno system identification and its application in the steel industry

M.H. Fazel Zarandi [a,*], R. Gamasaee [a], I.B. Turksen [b,c]

[a] Department of Industrial Engineering, Amirkabir University of Technology, P.O. BOX 15875-4413, Tehran, Iran
[b] Department of Industrial Engineering, TOBB University of Economics and Technology, Sogutozu, Ankara, Turkey
[c] Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, Ontario, Canada M5S2H8

ARTICLE INFO

Article history:
Received 17 April 2011
Received in revised form 19 August 2011
Accepted 19 October 2011
Available online 25 October 2011

Keywords:
IT2F c-regression clustering
Structure identification
Gaussian mixture
Weighted least square
Multiple-regression
Steel industry

ABSTRACT

This paper proposes a new type-2 fuzzy c-regression clustering algorithm for the structure identification phase of Takagi–Sugeno (T–S) systems. We present uncertainties with fuzzifier parameter "m". In order to identify the parameters of interval type-2 fuzzy sets, two fuzzifiers "m_1" and "m_2" are used. Then, by utilizing these two fuzzifiers in a fuzzy c-regression clustering algorithm, the interval type-2 fuzzy membership functions are generated. The proposed model in this paper is an extended version of a type-1 FCRM algorithm [25], which is extended to an interval type-2 fuzzy model. The Gaussian Mixture model is used to create the partition matrix of the fuzzy c-regression clustering algorithm. Finally, in order to validate the proposed model, several numerical examples are presented. The model is tested on a real data set from a steel company in Canada. Our computational results show that our model is more effective for robustness and error reduction than type-1 NFCRM and the multiple-regression.

1. Introduction

Because of the uncertain nature of many real world problems, the decision making for them is done based on uncertain information [44]. In such situations, fuzzy set theory can be utilized to model and solve problems with vague information. In order to model a problem with fuzzy set theory, two steps should be considered: system identification and fuzzy reasoning [19].

System identification is one of the main steps in every mathematical modeling [27]. It consists of three phases: structure identification, parameter tuning, and model validation [19]. In fuzzy system modeling, structure identification is related to the process of selecting relevant inputs, defining membership functions, and determining number of rules [19]. The parameter tuning phase determines the parameters of the inference system. The model validation tests the accuracy of the model [19]. The method used in the structure identification phase depends on the reasoning phase.

The reasoning phase of fuzzy modeling is divided into two main methods: Mamdani [28] and Takagi–Sugeno (T–S) [38] inference methods. In Mamdani's systems, all variables in consequents and antecedents have linguistic variables. In contrast, T–S systems have linguistic variables not in the consequent part, but in their antecedents. On the other hand, the consequent of a T–S system is a regression function. Hence, they require different reasoning and structure identification techniques.

Data clustering is one of the main methods of structure identification. It is an unsupervised technique to put similar data in a cluster [11]. Bezdek introduces fuzzy c-means clustering (FCM), for Mamdani systems. FCM has the constraint that the

* Corresponding author. Tel.: +98 21 645 45378; fax: +98 21 641 3969.
E-mail addresses: zarandi@aut.ac.ir (M.H. Fazel Zarandi), gamasaee@aut.ac.ir (R. Gamasaee), turksen@mie.utoronto.ca (I.B. Turksen).

0020-0255/$ - see front matter
doi:10.1016/j.ins.2011.10.015

summation of memberships of a data point in all clusters has to be one [3]. For structure identification phase of "T–S" system, type-1 FCRM method is proposed by Hathaway and Bezdek [17] (FCM is also used for T–S systems, but FCRM is a more suitable method for these kinds of systems). Type-1 FCRM is a fuzzy c-regression clustering algorithm in which the shapes of clusters are hyper planes. In contrast, in FCM algorithm, the shape of clusters is hyper-spheres [20,25]. In FCM, cluster centers are calculated to represent clusters, while in type-1 FCRM hyper planes play the role of cluster centers. Hyper planes are regression functions, which have inputs and one output.

There are several clustering techniques in the literature. The Gustafson–Kessel clustering algorithm uses the inverse of the fuzzy covariance matrix for computing the Mahalanobis distance [16]. Another clustering method is the Gath–Geva clustering algorithm, which uses adaptive distance norm [15]. A possibilistic approach to clustering has been introduced by Krishnapuram and Keller [22]. In this method the unity limitation of the summation of memberships of a data point in all clusters was removed. Sugeno and Yasukawa introduce the structure identification in which first the memberships of consequents are determined. Then, by projecting the memberships of consequents onto the input space the memberships of premises are obtained [37]. All these methods are used for type-1 fuzzy systems.

However, type-2 fuzzy systems are used to model the problems with a high level of vagueness in their information. Zadeh introduced type-2 fuzzy logic as an extension of type-1 fuzzy [43]. When determining the precise memberships is very difficult, the type-2 fuzzy sets have some advantages. They can handle the uncertainty of the information more efficiently than type-1 fuzzy sets.

For structure identification of this type of fuzzy system different methods have been introduced in the literature. For more information one can refer to [14,33–36,10,18]. In order to generate a rule based system, Aliev et al. proposed a type-2 version of FCM [1]. Although there are some works in literature considering type-2 fuzzy, there is no research on type-2 NFCRMA (a novel fuzzy c-regression model clustering algorithm presented in [25]). NFCRMA is an efficient clustering method for type-1 fuzzy T–S systems. It produces accurate results. However, it is unable to handle the high degree of uncertainty in real problems. Type-2 fuzzy systems can help to model these types of problems.

In this paper, a new type-2 fuzzy c-regression clustering algorithm for T–S system identification is proposed for steel industry. This algorithm is based on the novel type-1 fuzzy c-regression model clustering algorithm NFCRMA [25]. Type-2 fuzzy sets are used to control the higher degree of uncertainty that type-1 NFCRMA cannot handle. The main contribution of this paper is that we extend the method presented in [25] to an interval type-2 fuzzy model. The focus of our model is on fuzzifier m. By defining two fuzzifiers m_1 and m_2 the interval type-2 fuzzy clustering model is constructed [18]. The Gaussian mixture distribution is also used for defining memberships of partition matrix. The weighted least square model is used to calculate the regressions' coefficients in the consequents of T–S rules.

The rest of this paper is organized as follows: In Section 2, background information is presented. Section 3 proposes the model for interval type-2 fuzzy system. In Section 4, several numerical examples are presented to validate the proposed model. In this section the proposed model, first, is validated by several well known data in literature, then, it is used on a real data set from a steel company in Canada. Finally, in Section 5, conclusion and future works are presented.

2. Background

In this section, we briefly review the type-1 fuzzy clustering for T–S systems (FCRMA). Then, the interval type-2 fuzzy c-means algorithm is explained. Finally, the concept of Gaussian mixture distribution is illustrated.

2.1. Type-1 fuzzy clustering for T–S systems (NFCRMA)

As mentioned before, in the FCRMA clustering the shapes of clusters are hyper planes. The regression function in FCRM is presented in (1) [25].

$$y_k = f^i(x_k, \theta_i) = a_1^i x_{k1} + a_2^i x_{k2} + \cdots + a_M^i x_{kM} + b_0^i = [x_k 1].\theta_i^T, \quad i = 1, 2, \ldots, c \tag{1}$$

where, $x_k = x_{k1}, \ldots, x_{km}$ is the kth input vector and $\theta_i = \left[a_1^i, \ldots, a_M^i, b_0^i\right]$ is the coefficient of regression model.

The objective function in this algorithm is the same as FCM, but the distance between data pair to cluster representative is defined by Eq. (2). The aim of the algorithm is to minimize the objective function, represented in (3). Eq. (4) shows that in FCM and FCRM the summation of the memberships of a data point in all clusters has to be one [25].

$$d_{ik}(\theta_i) = |y_k - [x_k 1].\theta_i^T| \tag{2}$$

$$J_m(U, \theta) = \sum_{k=1}^{n} \sum_{i=1}^{c} (\mu_{ik})^m (y_k - [x_k 1].\theta_i^T)^2 = \sum_{k=1}^{n} \sum_{i=1}^{c} (\mu_{ik})^m \left(y_k - \sum_{j=1}^{M+1} \theta_{ij} x_{kj}^{\wedge}\right)^2 \tag{3}$$

where, $x^{\wedge}k = [x_k, 1]$, $m \in (1, \infty)$, is the fuzzy weighting exponent, and $\mu_{ik} \in [0, 1]$ is the fuzzy membership degree of kth data pair belonging to the ith cluster.

$$\sum_{i=1}^{c} \mu_{ik} = 1, \quad k = 1, 2, \ldots, n \tag{4}$$