Information Sciences 181 (2011) 4273–4286

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Entropy, similarity measure of interval-valued intuitionistic fuzzy sets and their applications

Cui-Ping Wei [a,*], Pei Wang [b], Yu-Zhong Zhang [a]

[a] College of Operations and Management, Qufu Normal University, Rizhao, Shandong 276826, PR China
[b] School of Management and Economics, Beijing Institute of Technology, Beijing 100086, PR China

ARTICLE INFO

Article history:
Received 17 September 2009
Received in revised form 13 January 2011
Accepted 1 June 2011
Available online 6 June 2011

Keywords:
Interval-valued intuitionistic fuzzy set
Entropy
Similarity measure

ABSTRACT

In this paper we propose an entropy measure for interval-valued intuitionistic fuzzy sets, which generalizes three entropy measures defined independently by Szmidt, Wang and Huang, for intuitionistic fuzzy sets. We also give an approach to construct similarity measures using entropy measures for interval-valued intuitionistic fuzzy sets. In particular, the proposed entropy measure for interval-valued intuitionistic fuzzy sets can yield a similarity measure. Several illustrative examples are given to demonstrate the practicality and effectiveness of the proposed formulas. We apply the similarity measure to solve problems on pattern recognitions, multi-criteria fuzzy decision making and medical diagnosis.

1. Introduction

1.1. Entropies and similarity measures of IFSs

Since Zadeh [33] introduced fuzzy sets, a lot of generalized forms have been proposed, among which there are Intuitionistic Fuzzy Sets (IFSs) [1], interval-valued intuitionistic fuzzy sets (IVIFSs) [2], vague sets [9], R-fuzzy sets [32] and Interval-Valued Fuzzy Sets (IVFSs) [34]. From the references [4,7,8], we know that IVFS theory is equivalent to IFS theory, which in its turn is equivalent to Vague Set theory, and IVIFS theory extends IFS theory.

As two important topics in the theory of fuzzy sets, entropy measures and similarity measures of IFSs have been investigated widely by many researchers from different points of view. Burillo and Bustince [3] introduced the notions of entropy of IVFSs and IFSs to measure the degree of intuitionism of an IVFS or IFS. Szimidt and Kacprzyk [22] proposed a nonprobabilistic-type entropy measure with a geometric interpretation of IFSs. Hung and Yang [14] gave their axiomatic definitions of entropy of IFSs and IVFSs by exploiting the concept of probability. Many authors also proposed different entropy formulas for IFS [27,28], IVFS [36–38] and Vague Set [11,12,39].

The similarity measures of IFSs are used to estimate the degree of similarity between two IFSs. Szimidt and Kacprzyk [21] defined a similarity measure using a distance measure which involves both similarity and dissimilarity. Expanding upon this work, Szimidt and Kacprzyk in [25] considered a family of similarity measures and compared with some existing similarity measures. Hong and Kim [10], Hung and Yang [15], Xu [31] defined independently some similarity measures based on different distance measures for IFSs. For the similarity measures of IFSs and their positive aspects, we refer to [17,31].

On the relationship between entropies and similarity measures of IFSs, Zeng and Guo [35] proved that some similarity measures and entropies of IVFSs can be deduced by normalized distances of IVFSs based on their axiomatic definitions. Zeng

* Corresponding author.
E-mail address: wei_cuiping@yahoo.com.cn (C.-P. Wei).

0020-0255/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.ins.2011.06.001

and Li [36], Zhang et al. [38] showed that similarity measures and entropies of IVFSs can be transformed by each other. Zhang and Yu et al. [37] put forward some entropy formulas of IFSs according to the relationship between entropies and similarity measures of IFSs.

1.2. Related work on entropies and similarity measures of IVIFSs

The entropy and similarity measure of IFSs have been applied widely in decision making [6,20,21] and pattern recognition [16,19,30]. However, in these applications, due to the increasing complexity of the social-economic environment and a lack of knowledge or data about the problem domains, the decision information may be provided with IVIFSs, which are characterized by membership functions and non-membership functions whose values are intervals, instead of real numbers. Therefore, it is highly necessary and important to study the entropy and similarity measure of IVIFSs. However, to our knowledge, not much research has been done in these respects. The followings are some of the research findings.

Liu et al. [18] proposed a set of axiomatic requirements for entropy measures of IVIFSs, which extends Szmidt and Kacprzyk's axioms formulated for entropy of IFSs [22]. Xu [31] generalized some formulas of similarity measures of IFSs to IVIFSs, which are based on the distance measures for IVIFSs. In some of Xu's similarity measures, he took into account the maximum of the absolute deviations between the corresponding endpoints of membership intervals (respective, non- membership intervals) of each element in two IVIFSs. As a result these measures cannot distinguish the similarity degree for some IVIFSs and may lead to counter-intuitive results.

In this paper, we propose a formula to calculate the entropy of an IVIFS on the basis of the argument on the relationship among the entropies of IFSs given in [12,22,28]. We also establish a method to construct similarity measures of IVIFSs by entropy measures of IVIFSs, by which we relate a degree of similarity between two IVIFSs *A* and *B* to the entropy of an IVIFS given by *A* and *B*. In particular, we give a formula of a similarity measure of IVIFSs by the proposed entropy formula of IVIFSs. Compared with Xu's similarity measure, this one is different and can overcome some drawbacks of counter-intuition. Moreover, even in the case of IFSs, the reduction of the proposed similarity measure of IVIFSs to IFSs is new.

The structure of this paper is as follows. Section 2 reviews some concepts of IFSs and IVIFSs. Section 3 proves the entropy formulas of IFSs proposed respectively in [12,22,28] are equal and generalizes these formulas to IVIFSs. Section 4 investigates the relationship between the entropy and similarity measure of IVIFSs and proves that similarity measures of IVIFSs can be constructed by entropy measures of IVIFSs based on the axiomatic definition. In particular, we give a similarity measure of IVIFSs according to the entropy formula of IVIFSs defined in Section 3. This is followed by applications of the proposed similarity measure to pattern recognition and multicriteria fuzzy decision making. This paper is concluded in Section 6.

2. Preliminaries

Definition 2.1 [1]. Let *X* be a universe of discourse. An intuitionistic fuzzy set in *X* is an object having the form:

$$A = \{\langle x, u_A(x), v_A(x)\rangle | x \in X\}, \tag{1}$$

where

$$u_A : X \to [0,1], \quad v_A : X \to [0,1]$$

with the condition

$$0 \leqslant u_A(x) + v_A(x) \leqslant 1, \quad \forall x \in X.$$

The numbers $u_A(x)$ and $v_A(x)$ denote the degree of membership and non-membership of x to *A*, respectively.

For convenience of notations, we abbreviate "intuitionistic fuzzy set" to IFS and denote by IFS (X) the set of all the IFSs in *X*.

For each IFS *A*in *X*, we call $\pi_A(x) = 1 - u_A(x) - v_A(x)$ the intuitionistic index of *x* in *A*, which denotes the hesitancy degree of *x* to *A*. The complementary set A^C of *A* is defined as

$$A^C = \{\langle x, v_A(x), u_A(x)\rangle | x \in X\}. \tag{2}$$

Consider that, sometimes, it is not approximate to assume that the membership degrees for certain elements of an IFS are exactly defined, but a value range can be given. In such cases, Atanassov and Gargov [2] introduced the following notions of interval-valued intuitionistic fuzzy sets:

Definition 2.2 [2]. Let *X* be a universe of discourse and int (0,1) denote all closed subintervals of the interval [0,1]. An interval-valued intuitionistic fuzzy set *A* in *X* is an object having the form:

$$A = \{\langle x, u_A(x), v_A(x)\rangle | x \in X\}, \tag{3}$$

where

$$u_A : X \to \text{int}(0,1), v_A : X \to \text{int}(0,1),$$