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A lattice matrix method for hyperspectral image unmixing

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ABSTRACT

In this manuscript we propose a method for the autonomous determination of endmembers in hyperspectral imagery based on recent theoretical advancements on lattice auto-associative memories. Given a hyperspectral image, the lattice algebra approach finds in a single-pass all possible candidate endmembers from which various affinely independent sets of final endmembers may be derived. In contrast to other endmember detection methods, the endmembers found using two dual canonical lattice matrices are geometrically linked to the data set spectra. The mathematical foundation of the proposed method is first described in some detail followed by application examples that illustrate the key steps of the proposed lattice based method.

1. Introduction

The high spectral resolution produced by current hyperspectral imaging devices facilitates identification of fundamental materials that make up a remotely sensed scene and thus supports discrimination between them. A pixel of a hyperspectral image physically represents a surface region on the ground comprising several square meters. Thus, a hyperspectral image pixel can have all or parts of many different natural or man-made objects in it. The collection of measured reflectances associated with the pixel is called the *spectrum* of the pixel. It is, therefore, useful to know the percentage of different fundamental object parts that are most represented in the spectrum of a given pixel. The most widely used spectral mixing model is the linear mixing model, which assumes that the observed reflectance spectrum of a given pixel is a linear combination of a small number of unique constituent signatures known as *endmembers* [1]. In various applications, hyperspectral image segmentation and analysis takes the form of a pattern recognition problem as the segmentation problem reduces to matching the spectra of the hyperspectral image to predetermined signatures stored in a spectral library. In many cases, however, endmembers cannot be determined in advance and must be selected from the image directly by identifying the pixel spectra that are most likely to represent the fundamental materials. This comprises the *autonomous endmember detection problem*. Unfortunately, the spatial resolution of a sensor makes it often unlikely that any pixel is composed of a single endmember. Thus, the determination of endmembers becomes a search for image pixels with the least contamination from other endmembers. These are also referred to as *pure pixels*. The pure pixels exhibit *maximal* or *minimal* reflectance in certain spectral bands and correspond to vertices of a high-dimensional simplex that, hopefully, encloses most if not all pixel spectra.

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In this paper we assume the *constrained linear mixing model* based on the fact that points on a simplex can be represented as a linear sum of the vertices that determine the simplex. The mathematical equations of the model and its constraints are given by

$$\mathbf{x} = S\mathbf{a} + \mathbf{r} = \sum_{i=1}^{m} a_i \mathbf{s}^i + \mathbf{r};$$
$$\sum_{i=1}^{m} a_i = 1 \text{ and } a_i \geqslant 0 \ \forall i, \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the measured spectrum of an image pixel, $S = (\mathbf{s}^1, \ldots, \mathbf{s}^m)$ is an $n \times m$ matrix whose columns are the m endmember spectra assumed to be affinely independent, the entries of $\mathbf{a} = (a_1, \ldots, a_m)^t$ are the corresponding abundances or fractions of the endmember spectra present in $\mathbf{x}$, and $\mathbf{r}$ represents a noise vector.

Endmembers may be obtained from spectral libraries for certain specific materials, or automatically from the image by a variety of techniques [5,7,28,37,39,40]. Autonomous endmember detection has received wide attention since signatures of various objects that may be present in an image are unknown before hand. Boardman [5] uses the framework of the geometry of convex sets to identify the $m+1$ endmembers as the vertices of the smallest simplex that bounds the measured data. However, the simplex vertices need not be image pixels and, hence, need not coincide with actual image data. Winter's N-FINDR method [39,40] is based on inflating a simplex within the data set to determine the largest simplex inscribed within the data. This algorithm is computationally intensive since individual pixels need to be examined and simplex volume recalculated for each image pixel. Recent methods proposed by Nascimento-Bioucas [28] and Chang et al. [7] offer similar or faster performance with respect to the N-FINDR method and are based on convex optimization techniques [6]. The autonomous endmember determination method proposed in this paper is also fast and carries little computational overhead. The method is derived from examining a lattice based auto-associative memory that stores the hyperspectral image cube. Graña et al., were the first to propose the use of lattice based auto-associative memories for autonomous endmember determination [15] as well as an evolutionary based strategy for endmember discrimination [16]. In the first approach, related to the present work, they employed the notion of morphological independence which does not necessarily lead to finding an affinely independent set of vectors that in some sense provides a maximal simplex within the data set. Furthermore, Graña's algorithm requires the user to choose a starting pixel and different starting pixels can produce different results. Improvements of algorithms using the preceding approach are based on recent discoveries of algebraic properties inherent in lattice based auto-associative memories [34] but the endmembers obtained need not be related with the hyperspectral image. The WM method described here differs from those described by Graña [17,18] and Myers [26] as the endmembers we obtain have a geometrical relationship to the pixels of the hyperspectral image under consideration. Our method will always provide the same sets of candidate endmembers based on theoretical facts given in this paper. To validate our proposed method, a brief comparison is made against two new approaches based on convex optimization, namely vertex component analysis [28] and the minimal volume enclosing simplex [7].

The paper is organized as follows. Section 2 introduces the reader to background concepts on binary lattice operations with numbers, vectors and matrices. Lattice associative memories (LAMs) and their fundamental properties are discussed in Section 3. The following three sections are devoted to the mathematical foundation that guarantees the correctness of the proposed method. Specifically, Section 4 establishes the geometric description of the fixed point set of lattice auto-associative memories. Section 5 establishes the relationships between the hyperspectral data cube, the corresponding LAMs and their fixed point set. In Section 6, we present the theoretical results needed to prove the affine independence of the sets (or proper subsets) of scaled column vectors derived from LAMs. Endmember determination using the WM method and constrained linear unmixing of hyperspectral images is presented in Section 7. Finally, a brief discussion about our proposed method and some conclusions are provided in Section 8.

2. Lattice theory fundamentals

The computational concepts for the associative neural networks used in this manuscript are governed by the *bounded lattice ordered group* $(\mathbb{R}_{\pm\infty}, \vee, \wedge, +, +')$, where $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}_{\pm\infty} = \mathbb{R} \cup \{-\infty, \infty\}$, $\vee$ and $\wedge$ denote the binary operation of maximum and minimum, respectively, + denotes addition, and $+'$ denotes the dual operation of + defined by $a +' b = a + b$ for any $a \in \mathbb{R}$. If $a \in \mathbb{R}_{\pm\infty}$, then its *additive conjugate* is given by $a^* = -a$.

Unless stated otherwise, a vector $\mathbf{x} \in \mathbb{R}^n_{\pm\infty}$ is always viewed as a column vector, i.e., $\mathbf{x} = (x_1, \ldots, x_n)^t$, where $x_i \in \mathbb{R}_{\pm\infty}$ for $i = 1, \ldots, n$ and t denotes the transpose. Scalar addition of a vector $\mathbf{x} \in \mathbb{R}^n_{\pm\infty}$ is defined componentwise. That is, if $a \in \mathbb{R}_{\pm\infty}$, then $a + \mathbf{x} = (a + x_1, \ldots, a + x_n)^t$. The *conjugate* of $\mathbf{x} \in \mathbb{R}^n_{\pm\infty}$ is defined as $\mathbf{x}^* = -\mathbf{x}^t$. Given two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n_{\pm\infty}$, then the *maximum* and *minimum* of $\mathbf{x}$ and $\mathbf{y}$, denoted by $\mathbf{x} \vee \mathbf{y}$ and $\mathbf{x} \wedge \mathbf{y}$, respectively, are defined componentwise as $(\mathbf{x} \vee \mathbf{y})_i = x_i \vee y_i$ and $(\mathbf{x} \wedge \mathbf{y})_i = x_i \wedge y_i$ for $i = 1, \ldots, n$. We note that the following *duality* De Morgan's type identities hold: $\mathbf{x} \vee \mathbf{y} = (\mathbf{x}^* \wedge \mathbf{y}^*)^*$ and $\mathbf{x} \wedge \mathbf{y} = (\mathbf{x}^* \vee \mathbf{y}^*)^*$. The inequalities $\mathbf{x} \leqslant \mathbf{y}$ and $\mathbf{x} < \mathbf{y}$ mean that $x_i \leqslant y_i$ and $x_i < y_i$, respectively, where $i = 1, \ldots, n$. Thus, if $\mathbf{u} = \mathbf{x} \vee \mathbf{y}$ and $\mathbf{v} = \mathbf{x} \wedge \mathbf{y}$, then $\mathbf{v} \leqslant \mathbf{u}$.

As our application domain concerns only real valued vectors, we restrict our discussion to sets of vectors $X = \{\mathbf{x}^1, \ldots, \mathbf{x}^k\} \subset \mathbb{R}^n_{\pm\infty}$ for which $\mathbf{x}^\xi \in \mathbb{R}^n$ and $\xi \in K$ where $K = \{1, \ldots, k\}$ is a finite set of positive integers. With this