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Generalized fuzzy rough sets determined by a triangular norm

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ABSTRACT

The theory of rough sets has become well established as an approach for uncertainty management in a wide variety of applications. Various fuzzy generalizations of rough approximations have been made over the years. This paper presents a general framework for the study of *T*-fuzzy rough approximation operators in which both the constructive and axiomatic approaches are used. By using a pair of dual triangular norms in the constructive approach, some definitions of the upper and lower approximation operators of fuzzy sets are proposed and analyzed by means of arbitrary fuzzy relations. The connections between special fuzzy relations and the *T*-upper and *T*-lower approximation operators of fuzzy sets are also examined. In the axiomatic approach, an operator-oriented characterization of rough sets is proposed, that is, *T*-fuzzy approximation operators are defined by axioms. Different axiom sets of *T*-upper and *T*-lower fuzzy set-theoretic operators guarantee the existence of different types of fuzzy relations producing the same operators. The independence of axioms characterizing the *T*-fuzzy rough approximation operators is examined. Then the minimal sets of axioms for the characterization of the *T*-fuzzy approximation operators are presented. Based on information theory, the entropy of the generalized fuzzy approximation space, which is similar to Shannon's entropy, is formulated. To measure uncertainty in *T*-generalized fuzzy rough sets, a notion of fuzziness is introduced. Some basic properties of this measure are examined. For a special triangular norm $T = \min$, it is proved that the measure of fuzziness of the generalized fuzzy rough set is equal to zero if and only if the set is crisp and definable.

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1. Introduction

The theory of rough sets was originally proposed by Pawlak in 1982 [25,26] as a mathematical approach to handle imprecision, vagueness, and uncertainty in data analysis. It has recently received wide attention in real-life applications and theoretical research. By using the concepts of lower and upper approximations in rough set theory, knowledge hidden in information systems may be unraveled and expressed in the form of decision rules. There are at least two methods for the development of this theory, namely, the constructive and axiomatic approaches [44]. In constructive methods, lower and upper approximations are constructed from the primitive notions, such as binary relations on a universe of discourse [17,25,26,29,30,43,44,53,52], neighborhood systems [13,15,18] and Boolean algebras [14,24]. On the other hand, by treating abstract operators as primitive notions, axiomatic methods focus on algebraic systems for the theory of rough sets. In this

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approach, a set of axioms is used to characterize approximation operators that are the same as the ones produced by using the constructive approach.

The initiation and majority of studies on rough sets have been concentrated on the constructive approach. In Pawlak's rough set model [25], an equivalence relation is a key and primitive notion. This equivalence relation, however, seems to be a very stringent condition that may limit the application of the rough set model. To solve this problem, many authors have generalized the notion of approximation operators by using nonequivalence binary relations [11,30,37,45,46]. Moreover, rough sets can also be generalized to the fuzzy environment and the results are called rough fuzzy sets and fuzzy rough sets [3,7,9,10,18–23,32–34,47,51].

Recently, much effort has been made in the axiomatic approach, particularly from the operator-oriented point of view. Lin and Liu [16] proposed six axioms on a pair of abstract operators on the power set of the universe of discourse within the framework of topological space, under which there exists an equivalence relation reproducing the lower and upper approximation operators obtained by the constructive approach. Similar result was also stated earlier by Wiweger [36]. Zakowski [50] studied a set of axioms on approximation operators. Comer [5,6] investigated axioms on approximation operators in relation to cylindric algebras. However, all of these studies are restricted to the Pawlak rough set algebra defined by equivalence relations. In [31], Thiele analyzed the axiomatic characterizations of crisp approximation operators. Wybraniec-Skardowska [40] examined many axioms on various classes of approximation operators, and different constructive methods were suggested to produce such approximation operators. Mordeson [21] investigated the axiomatic characterization of approximation operators defined by covers. The most important axiomatic studies for crisp rough sets were reported by Yao and Lin [43,44,49], where various crisp rough set algebras were characterized by using different sets of axioms. The axiomatic approach has also been extended to the study of approximation operators in the fuzzy environment. Morsi and Yakout [22] studied a set of axioms on fuzzy rough approximation operators defined by a triangular norm and a residual implication. But their studies were restricted to the fuzzy rough set algebras constructed by fuzzy equivalence relations. Thiele [32–34] investigated axiomatic characterizations of fuzzy rough approximation operators and rough fuzzy approximation operators within modal logic for fuzzy diamond and box operators. Based on a fuzzy similarity relation, Radzikowska and Kerre [27] defined a broad family of so called $(\mathscr{I},\mathscr{T})$-fuzzy rough sets which is determined by an implicator $\mathscr{I}$ and a triangular norm $\mathscr{T}$. However, the properties and axiomatic characterization of $(\mathscr{I},\mathscr{T})$-fuzzy rough sets corresponding to an arbitrary fuzzy relation or a special fuzzy relation have not been studied. Wu et al. [37–39] examined many axioms on various classes of rough fuzzy and fuzzy rough approximation operators. Mi and Zhang [20] discussed axiomatic characterization of a pair of dual lower and upper fuzzy approximation operators based on a residual implication. However, all of the aforementioned studies have not solved the important problem of the independence and minimization of the axioms for approximation operators. Recently, Yang and Li [41,42] presented the minimization of axioms for generalized crisp rough approximation operators and fuzzy rough approximation operators defined by Dubois and Prade [7].

Information theory, originally developed by Shannon [28] in the study of communication, has been a useful methodology for characterizing the information content in various models and applications in many diverse fields. Attempts have been made to use Shannon's entropy to measure uncertainty in rough set theory [1,8,35,48]. Recently, Chakrabarty et al. [4] introduced a measure of fuzziness in rough sets. Their measure is based on a special index of fuzziness of a fuzzy set. Wierman [35] and Liang et al. [12] introduced measure of uncertainty for Pawlak's rough set theory. Mi et al. [18] introduced an uncertainty measure in partition-based fuzzy rough set.

In this paper, we propose a general framework for the study of T-fuzzy rough approximation operators in which both the constructive and axiomatic approaches are used, and present a novel uncertainty measure of the generalized fuzzy rough sets. By using a pair of dual triangular norms in the constructive approach, some definitions of upper and lower approximation operators of fuzzy sets are proposed and analyzed. Our definition of approximations is derived from an arbitrary fuzzy relation over two universes of discourse, which can be used in approximation reasoning under fuzzy environment. The axiomatic characterizations of generalized fuzzy approximation operators are also examined. Then the minimal sets of axioms for the characterization of the T-fuzzy approximation operators are obtained. Thus, in the case of finite universe of discourse, we solve the open problem (concerning a complete operator-oriented characterization of Lukasiewicz fuzzy rough sets) proposed by Radzikowska and Kerre in [27]. To measure uncertainty in T-generalized fuzzy rough sets, a notion of fuzziness is introduced, which is demonstrated to be adequate for measuring the uncertainty of a fuzzy event in the fuzzy approximation space.

In Section 2, based on an arbitrary fuzzy relation, a pair of T-upper and T-lower fuzzy approximation operators is defined, and the properties of the T-fuzzy rough approximation operators are investigated. In Section 3, T-fuzzy rough sets are axiomatized by abstract operators. Various classes of fuzzy approximation operators are characterized by different sets of axioms. The axiom set of the T-fuzzy rough approximation operators guarantee the existence of certain types of fuzzy relations producing the same operators. The independence of axioms for the T-fuzzy rough approximation operators is examined in Section 4, and the minimal sets of axioms of the approximation operators are then presented. Based on information theory, the entropy of a generalized fuzzy approximation space, which is similar to Shannon's entropy, is introduced in Section 5. In Section 6, a notion of fuzziness of the T-generalized fuzzy rough sets is defined as the entropy of rough belongingness. Some basic properties of this uncertainty measure are examined. Finally, in Section 7, we conclude the paper with a summary and an outlook for further research.

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