Information Sciences 181 (2011) 1116–1124

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

On averaging operators for Atanassov's intuitionistic fuzzy sets

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ARTICLE INFO

Article history:
Received 8 February 2010
Received in revised form 22 October 2010
Accepted 17 November 2010

Keywords:
Atanassov's intuitionistic fuzzy sets
Aggregation operators
Interval-valued fuzzy sets
OWA
Triangular norms

ABSTRACT

Atanassov's intuitionistic fuzzy set (AIFS) is a generalization of a fuzzy set. There are various averaging operators defined for AIFSs. These operators are not consistent with the limiting case of ordinary fuzzy sets, which is undesirable. We show how such averaging operators can be represented by using additive generators of the product triangular norm, which simplifies and extends the existing constructions. We provide two generalizations of the existing methods for other averaging operators. We relate operations on AIFS with operations on interval-valued fuzzy sets. Finally, we propose a new construction method based on the Łukasiewicz triangular norm, which is consistent with operations on ordinary fuzzy sets, and therefore is a true generalization of such operations.

1. Introduction

Since the introduction of fuzzy sets by Zadeh [39] many attempts have been made to generalize the notion of fuzzy sets. Among others, Zadeh introduced the idea of type-2 fuzzy sets and interval valued fuzzy sets [40], see also [18,41,42]. Later in [2], Atanassov introduced the idea of intuitionistic fuzzy sets (AIFS). Recently, several authors [23,25–30] have used AIFS in different applications. In [3,8,16] authors advanced the theory of operators and relations for intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets. Bustince and Burillo [8] and Deschrijver and Kerre [11] made theoretical development relating to composition of intuitionistic fuzzy relations.

In many decision making applications, it becomes necessary to aggregate several fuzzy sets, particularly when preference is expressed by fuzzy sets [6,7,22,37]. Weighted means and the Ordered Weighted Averaging functions (OWA) [37] have been applied to this problem, along with triangular norms, conorms and uninorms. Various aggregation functions are discussed in detail in [7,19,20,32].

In a recent series of papers [31,33–36,38,43] the authors defined some averaging aggregation functions for Atanassov's intuitionistic fuzzy sets, including weighted means, OWA and Choquet integrals. Their definitions are based on the operations of addition and multiplication for AIFS [3,4], which involve the product t-norm and its dual t-conorm. A problem with their definitions is that they do not lead to the standard aggregation operations on fuzzy sets in the limiting case (see Example 2).

On the other hand, as noticed in [5] and later developed in [8,10,14], the AIFS are mathematically equivalent to the interval-valued fuzzy sets (IVFS). There exist several papers that relate other extensions of fuzzy set theory to AIFS [9,17,24]. It is

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0020-0255/$ - see front matter
doi:10.1016/j.ins.2010.11.024

straightforward to define aggregation functions for IVFS, see, e.g. [38], by applying a fixed aggregation function to the ends of the membership interval, the membership and (transformed) non-membership degrees. Such functions are called representable in [15]. This approach is fully consistent with the limiting case of the ordinary fuzzy sets, but there are various non-representable AIFS aggregation functions, such as those presented in [12,13,15], and those discussed in this paper.

In this paper we develop an approach to extending aggregation operators to AIFS and IVFS, by using additive generators of the t-norm and t-conorm in the arithmetical operations for AIFS. We provide simple tools to construct both representable and non-representable extensions. We establish several interesting properties of the operators constructed by using Łukasiewicz t-norm and t-conorm in the operations for AIFS. Our approach will eliminate the need for complex and explicit constructions in [34–36,43].

The structure of this paper is as follows. We review operations on AIFS and IVFS in Section 2. In Section 3 we present several types of averaging aggregation functions on AIFS based on arithmetical operations on AIFS. In Section 4 we relate the mentioned aggregation functions to those defined for IVFS, and present several more general approaches to aggregation of AIFS. We will show that the use of Łukasiewicz t-norm and t-conorm in the definition of addition of AIFS guarantees consistency of aggregation of AIFS with aggregation of ordinary fuzzy sets. This section is then followed by conclusions.

2. Operations on AIFS and IVFS

We review several relevant concepts and highlight the correspondence between the notions in AIFS and IVFS [2].

Definition 1. An AIFS $\mathcal{A}$ on X is defined as $\mathcal{A} = \{\langle x, \mu_{\mathcal{A}}(x), \nu_{\mathcal{A}}(x)\rangle | x \in X\}$, where $\mu_{\mathcal{A}}(x)$ and $\nu_{\mathcal{A}}(x)$ are the degrees of membership and nonmembership of x in $\mathcal{A}$, which satisfy $\mu_{\mathcal{A}}(x),\ \nu_{\mathcal{A}}(x) \in [0,1]$ and $0 \leqslant \mu_{\mathcal{A}}(x) + \nu_{\mathcal{A}}(x) \leqslant 1$.

Definition 2. An IVFS $\mathcal{A}$ on X is defined as $\mathcal{A} = \{\langle x, [l_{\mathcal{A}}(x), r_{\mathcal{A}}(x)]\rangle | x \in X\}$, where $l_{\mathcal{A}}(x)$ and $r_{\mathcal{A}}(x)$ are the lower and upper ends of the membership interval, and satisfy $0 \leqslant l_{\mathcal{A}}(x) \leqslant r_{\mathcal{A}}(x) \leqslant 1$.

Obviously an ordinary fuzzy set can be written as $\{\langle x, \mu_{\mathcal{A}}(x), 1-\mu_{\mathcal{A}}(x)\rangle | x \in X\}$, or as $\{\langle x, [\mu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(x)]\rangle | x \in X\}$. AIFS can be represented by means of IVFS and vice versa.

To ease the notation we now suppress the dependence of the membership/non-membership values on x, i.e., we will consider Atanassov's intuitionistic fuzzy values (AIFV), the pairs $A = \langle \mu_A, \nu_A\rangle$, and IV fuzzy values (IVFV) $[l_A, r_A]$.

Several indices are used to characterize AIFV.

Definition 3. The *Score* and *Accuracy* of an AIFV A are defined by

$$Score(A) = \mu_A - \nu_A,$$
$$Accuracy(A) = \mu_A + \nu_A.$$

The degree of indeterminancy of A is

$$\pi(A) = 1 - (\mu_A + \nu_A).$$

We list the indices in Table 1 together with their counterparts in the IVFS representation. We use $\frac{l+r}{2}$ to denote the center of the interval $[l, r]$.

Definition 4 (*[7,19]*). An aggregation function $f : [0,1]^n \to [0,1]$ is a function non-decreasing in each argument and satisfying $f(0,\ldots,0) = 0$ and $f(1,\ldots,1) = 1$.

An aggregation function is idempotent if $f(t,t,\ldots,t) = t$ for all $t \in [0,1]$. This is equivalent to averaging behavior of an aggregation function, i.e., $\min(x) \leqslant f(x) \leqslant \max(x)$ for all $x \in [0,1]^n$.

Let us denote by L the lattice of non-empty intervals $L = \{[a,b] | (a,b) \in [0,1]^2,\ a \leqslant b\}$ with the partial order $\leqslant_L$ defined as $[a,b] \leqslant_L [c,d] \iff a \leqslant c$ and $b \leqslant d$. The top and bottom elements are respectively $1_L = [1,1]$, $0_L = [0,0]$. The corresponding

Table 1
Various indices and operations on AIFV and IVFV.

Index	AIFV representation	IVFV representation
Membership	$\langle \mu, \nu \rangle$	$[l, r] = [\mu, 1-\nu]$
Degree of indeterminancy	$\pi = 1 - (\mu + \nu)$	$r - l = length([l, r])$
Score	$\mu - \nu$	$r + l - 1 = 2(\frac{l+r}{2}) - 1$
Accuracy	$\mu + \nu = 1 - \pi$	$l - r + 1 = 1 - length([l, r])$