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Uncertainty measures for general Type-2 fuzzy sets

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ABSTRACT

Five uncertainty measures have previously been defined for interval Type-2 fuzzy sets (IT2 FSs), namely centroid, cardinality, fuzziness, variance and skewness. Based on a recently developed α-plane representation for a general T2 FS, this paper generalizes these definitions to such T2 FSs and, more importantly, derives a unified strategy for computing all different uncertainty measures with low complexity. The uncertainty measures of T2 FSs with different shaped *Footprints of Uncertainty* and different kinds of secondary membership functions (MFs) are computed and are given as examples. Observations and summaries are made for these examples, and a *Summary Interval Uncertainty Measure* for a general T2 FS is proposed to simplify the interpretations. Comparative studies of uncertainty measures for Quasi-Type-2 (QT2), IT2 and T2 FSs are also performed to examine the feasibility of approximating T2 FSs using QT2 or even IT2 FSs.

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1. Introduction

This paper is about uncertainty measures for Type-2 fuzzy sets (T2 FSs). In order to understand why one should be interested in such measures for T2 FSs, one must first recall some facts about uncertainty, uncertainty measures, and uncertainty measures for both type-1 (T1) and interval Type-2 (IT2) FSs.

Zadeh [100] points out that "*uncertainty is an attribute of information*," and introduced the general theory of uncertainty (GTU), "*because existing approaches to representation of uncertain information are inadequate for dealing with problems in which uncertain information is perception-based and is expressed in a natural language*." He also states that "*In GTU, uncertainty is linked to information through the concept of granular structure – a concept which plays a key role in human interaction with the real world* [31,95,101]."

It is necessary to quantify the uncertainty associated with fuzzy sets (FS) as they are used as granules in GTU, because, as Klir [37] points out, "*once uncertainty (and information) measures become well justified, they can very effectively be utilized for managing uncertainty and associated information. For example, they can be utilized for extrapolating evidence, assessing the strength of relationship between given groups of variables, assessing the influences of given input variables on given output variables, measuring the loss of information when a system is simplified, and the like.*"

Klir [37] and Harmanec [30] have developed three fundamental principles to guide the use of uncertainty measures under different circumstances:

(1) "The principle of minimum uncertainty, *which states that solutions with the least loss of information should be selected, can be used in simplification and conflict resolution problems.*"

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(2) "*The* principle of maximum uncertainty, *which states that a conclusion should maximize the relevant uncertainty within constraints given by the verified premises, is widely used within classical probability framework* [15,16,65]."
(3) "*The* principle of uncertainty invariance, *which states that the amount of uncertainty should be preserved in each transformation of uncertainty from one mathematical framework to another, is widely studied in the context of probability-possibility transformation* [24,36,38,76]."

Cross and Sudkamp [21] indicate that "*the quantification of the degree of uncertainty in a FS depends upon the type of uncertainty one is trying to measure and on the particular measure selected for that type of uncertainty*."

Among many uncertainty measures proposed for T1 FSs, the most frequently-used are centroid, cardinality, fuzziness (entropy), variance and skewness, which provide us with very useful characteristics of such FSs. For example, as Karnik and Mendel [33] pointed out, the centroid of a T1 FS can be viewed "*as analogous (not equal!) to the mean of a probability density function*." It is the most fundamental uncertainty measure for T1 FSs, and is used in the definitions of several other uncertainty measures, e.g., variance and skewness.

Regarding cardinality of a T1 FS, Dubois and Prade [22] state: [cardinality] "*is a natural tool for capturing the meaning of linguistic quantifiers* [89–91,94,96–98] *and provide satisfactory answers to queries pertaining to quantification, of the form 'How many X's are A', 'Are there more X's which are A than X's which are B', etc.*". Wygralak [85] pointed out that these queries "*occur in computing with words, communication with data bases and information/intelligent systems, modeling the meaning of imprecise quantifiers in natural language statements, decision-making in a fuzzy environment, analysis of grey tone images, clustering, etc.*".

Wenstop [75] showed how to use the centroid and the cardinality of T1 FSs to measure the distance between two T1 FSs, which allows one to find the FS that most resembles a target T1 FS A among a group of T1 FSs B_i ($i = 1,\ldots,N$). Bonissone [4,5] proposed a two-step approach to solve the same problem by simultaneously utilizing centroid, cardinality, fuzziness and skewness. As for other related measures, Bustince et al. [9] have showed how to define and construct Decreasing–Increasing (DI)-subsethood measures for T1 FSs.

For more than a decade, IT2 FSs have been applied in many areas, such as: decision making [53,64,68,79,87], time series forecasting [3,35,55], survey processing [2,45,55], document retrieval [6], speech recognition [51,102], noise cancellation [11,13,63], word modeling [45,59,80], clustering [66], control [1,10,12,13,23,28,41,43,49,50,67,83,84], wireless communication [42,69], web-shopping [27], linguistic summarization of database [61,62], etc. Because of their importance in so many applications, uncertainty [82,88] and related measures [7,8,78] of IT2 FSs have become interests for researchers. Wu and Mendel [82] showed how the Wavy-Slice Representation Theorem (WS RT)[1] of a T2 FS could be used to extend the above T1 uncertainty measures to IT2 FSs. Their work also explained how to compute these IT2 uncertainty measures. They have also shown [60,77,78] how the cardinality and centroid of an IT2 FS can be used in Computing With Words (CWW) [92,93].

Recently, there has been a growing interest in general T2 FSs and fuzzy logic systems (FLSs) [17–20,25,26,34,44,52,55,58,70–74,99]. Such FSs have more design degrees of freedom than do IT2 FSs, consequently, a general T2 FLS has the potential to out-perform an IT2 FLS. Just as an IT2 FLS involves type-reduction (TR) so does a general T2 FLS. Centroid TR is one popular form of TR; it requires computing the centroid of a T2 FS, which explains why this uncertainty measure for a general T2 FS has already been the subject of research.

Until very recently, there was no practical way to compute the centroid of a general T2 FS, because this computation requires computing the centroids of all of the embedded T2 FSs (these are defined in Section 2.1) of the general T2 FS. Enumerating all such sets is difficult and, depending upon how finely one discretizes the primary and secondary memberships of a T2 FS, there can be an extremely large number of such embedded T2 FSs.

To get around this, Greenfield et al. [26] proposed to compute the centroid of a small number of randomly selected embedded T2 FSs, instead of computing the centroids of all the embedded T2 FSs. Coupland [17] proposed to utilize the x-coordinate of the geometric centroid of the 3D MF of the T2 FS. John and Czarnecki [32] and Lucas et al. [47] proposed to use the centroids of all the vertical slices of the T2 FS.

Even more recently, an α*-plane RT* (also called Horizontal Slice or z-slice *RT*) of a T2 FS was introduced by several research groups independently, i.e., Liu [44], Wagner and Hagras [74], Tahayori et al. [72], and Chen and Kawase [14] (the term α*-plane representation* was first coined by Liu [44]). By applying the α*-plane RT*, Liu [44] showed that the centroid of a T2 FS can be obtained by taking the union of the centroids of all the α*-planes* of that T2 FS. Each α*-plane* centroid can be computed by the Karnik–Mendel (KM) or Enhanced KM (EKM) algorithms [34,55,57,81].

This paper generalizes Liu's centroid calculation to other T2 FS uncertainty measures, namely, cardinality, fuzziness, variance and skewness for general T2 FSs. Just as such uncertainty measures are used in T1 and IT2 applications, as explained above, we expect them also to be used in general T2 FS applications.

The rest of this paper is organized as follows: Section 2 provides background material about general T2 FSs; Section 3 defines the uncertainty measures for a general T2 FS, shows how they can be computed using the α*-plane RT*, and provides a summary of existing algorithms for computing uncertainty measures for an IT2 FS, because they are needed to compute the uncertainty measures for a general T2 FS; Section 4 gives examples of uncertainty measures for general T2 FSs for different Footprints of Uncertainty (*FOUs*) and secondary MFs; Section 5 provides quantitative interpretations and summaries for the

[1] The *WS RT* is also known as *embedded T2 FS* representation or the *Mendel–John Representation Theorem*, and is defined in Section 2.1.

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