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Random weighting estimation of parameters in generalized Gaussian distribution

SheSheng Gao [a,*], Zhihua Feng [a,b], Yongmin Zhong [c], Bijan Shirinzadeh [c]

[a] *School of Automatics, Northwestern Polytechnical University, Xi'an 710072, China*
[b] *The Second Academy, China Aerospace Science and Industry Corporation, Beijing 100854, China*
[c] *Department of Mechanical Engineering, Monash University, Vic. 3800, Australia*

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Abstract

This paper studies random weighting estimation of shape and scale parameters in generalized Gaussian distribution (GGD). An expression is established to describe the relationship between moments and parameters. The strong convergence for random weighting estimation of GGD parameters is also rigorously proved. Computational simulations and practical experiments are presented to demonstrate the efficacy for random weighting estimation of GGD parameters.

Keywords: Generalized Gaussian distribution; Random weighting estimation; Strong convergence

1. Introduction

Random weighting is an emerging computing method in statistics [7]. It has received considerable attention in the recent years, and has been widely studied for different problems [1–6]. However, there is little research to investigate random weighting estimation of parameters in generalized Gaussian distribution (GGD) (see Appendix A.1) and the related convergence properties.

Suppose that $X_1, X_2, \ldots, X_n$ are the random variables of an independent and identical distribution with an unknown distributed function F. Let $x_1, \ldots, x_n$ be the corresponding observed realizations. Further, we shall denote $\tilde{X}_n = (X_1, X_2, \ldots, X_n)$ and $\tilde{x}_n = (x_1, x_2, \ldots, x_n)$. Then, the random weighting process can be described as follows:

(i) Construct the sample (empirical) distribution function F_n from $\tilde{x}$, i.e.

$$F_n = \frac{1}{n}\sum_{i=1}^{n} X_i. \tag{1}$$

* Corresponding author.
E-mail address: gshshnpu@163.com (S. Gao).

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(ii) The random weighting estimation of the sample mean F_n is

$$H_n(x) = \sum_{i=1}^{n} V_i I_{(X_i \leqslant x)}, \tag{2}$$

where $I_{(X_i \leqslant x)}$ is the characteristic function, and random vector $(V_1, \ldots, V_n)$ obeys the Dirichlet distribution $D(1, \ldots, 1)$, i.e. $\sum_{i=1}^{n} V_i = 1$. A uniformly distributed density function of $(V_1, \ldots, V_n)$ can be defined as

$$f(V_1, \ldots, V_n) = (n-1)! \tag{3}$$

where $(V_1, V_2, \ldots, V_n) \in S_{n-1}$, and $S_{n-1} = \left\{(V_1, V_2, \ldots, V_n) : V_i \geqslant 0, i = 1, \ldots, n-1, \sum_{i=1}^{n-1} V_i \leqslant 1\right\}$.

This paper investigates the estimation of GGD parameters by using the random weighting method. An expression is established to describe the relationship between moments and parameters. The strong convergence for random weighting estimation of GGD parameters is also rigorously proved. Computational simulations and practical experiments have been conducted to comprehensively evaluate the performance for random weighting estimation of GGD parameters.

2. Random weighting estimation of GGD parameters

2.1. K-Order absolute moment of GGD parameters

The probability density function of GGD can be given as

$$f(x, \mu, \alpha, \beta) = \left[\frac{\alpha}{2\beta\Gamma(1/\alpha)}\right] \exp\left\{-\left[\frac{|x-\mu|}{\beta}\right]^{\alpha}\right\}, \quad -\infty < x < +\infty, \tag{4}$$

where μ, α and β are the mean, shape and scale parameters, respectively, and $\alpha > 0$. $\Gamma(z) = \int_0^{+\infty} e^{-t} t^{z-1}\, dt$ is Γ function (see Appendix A.2). $\beta = \sigma\sqrt{\Gamma(1/\alpha)/\Gamma(3/\alpha)}$, where σ is the standard deviation. The shape parameter α determines the decay speed for the density function of GGD:

- If $\alpha \to 0$, the limitation of the density function is the δ function (see Appendix A.3);
- If $\alpha = 1$, GGD corresponds to the Laplacian distribution (see Appendix A.4);
- If $\alpha = 2$, GGD corresponds to the Gaussian distribution;
- If $\alpha \to +\infty$, the limitation of GGD is a uniform distribution.

To discuss random weighting estimation of parameters α and β, we assume that $\tilde{x} = (x_1, x_2, \ldots, x_n)$ is a sample from the GGD collection with a given mean $\mu = 0$.

Since GGD is a symmetric distribution and its origin moments under any odd orders are zero, only the absolute moments are considered in random estimation of GGD parameters. When $\mu = 0$, by (4) the k-order absolute moment m_k can be written as

$$m_k = E(|X|^k) = \beta^k \frac{\Gamma((k+1)/\alpha)}{\Gamma(1/\alpha)} \quad (k = 1, 2, \ldots). \tag{5}$$

To obtain the random weighting estimation for parameter α, the statistic function T is constructed as

$$T = \frac{[E(|X|^k)]^l [E(|X|^m)]^n}{[E(|X|^p)]^q [E(|X|^r)]^s}, \tag{6}$$

where k, m, p and r are positive integers. l, n, q and s are non-negative integers. When $kl + mn = pq + rs$, T contains parameter α only. Thus, T may be written as

$$T = \frac{(m_k)^l (m_m)^n}{(m_p)^q (m_r)^s} = \frac{\Gamma^l\left(\frac{k+1}{\alpha}\right)\Gamma^n\left(\frac{m+1}{\alpha}\right)}{\Gamma^q\left(\frac{p+1}{\alpha}\right)\Gamma^s\left(\frac{r+1}{\alpha}\right)} \Gamma^{q+s-l-n}\left(\frac{1}{\alpha}\right) = F(\alpha). \tag{7}$$