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# Determining objective weights with intuitionistic fuzzy entropy measures: A comparative analysis

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ABSTRACT

In this paper, we propose a new objective weighting method that employs intuitionistic fuzzy (IF) entropy measures to solve multiple-attribute decision-making problems in the context of intuitionistic fuzzy sets. Instead of traditional fuzzy entropy, which uses the probabilistic discrimination of attributes to obtain attribute weights, we utilize the IF entropy to assess objective weights based on the credibility of the input data. We examine various measures for IF entropy with respect to hesitation degree, probability, non-probability, and geometry to calculate the attribute weights. A comparative analysis of different measures to generate attribute rankings is illustrated with both computational experiments as well as analyses of Pearson correlations, Spearman rank correlations, contradiction rates, inversion rates, and consistency rates. The experimental results indicate that ranking the outcomes of attributes not only depends on the type of IF entropy measures but is also affected by the number of attributes and the number of alternatives.


## 1. Introduction

In multiple-attribute decision-making (MADM) analysis, a decision maker (DM) must evaluate alternatives with respect to each attribute, address attribute weights, and select the best result from the generated set of alternatives. The MADM approach provides an effective way to select among non-commeasurable and conflicting attributes. Some representative methods, such as the simple additive weighting method (SAW), the analytic hierarchy process (AHP), and the technique for order preference by similarity to ideal solution (TOPSIS), have been developed to solve MADM problems [11,18,28]. Numerical data cannot be used to accurately model real-life circumstances because human judgments and preferences include fuzziness and vagueness. For this reason, research has extended the use of fuzzy set theory proposed by Zadeh [39] to MADM methods [7,9,20,37]. Using the notion of a conventional fuzzy set, Atanassov [2] introduced the concept of an intuitionistic fuzzy set (IFS), which is a generalization of the fuzzy set, to provide additional information about indeterminacy degrees. Since IFSs can adequately measure the decision-making process of human beings and cope with incomplete information, the merits of IFSs have been applied in various fields, including logical reasoning [19], pattern recognition [16], and decision-making [36]. In particular, Li [22] and Lin et al. [24] used a linear programming model to assess the optimal attribute weights in MADM. Liu and Wang [25] adopted the max–min, max-center, and max–max methods, which were introduced by Chen and Tan [8] and Hong and Choi [14], to yield weights within an intuitionistic fuzzy context. Xu [35] utilized similarity measures to solve MADM problems. Yu et al. [38] presented a fuzzy optimization method to manage MCDM problems in terms of the inclusion

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degree of IFSs. Li et al. [23] proposed a fractional programming method based on the TOPSIS to handle multiple-attribute group decision-making problems using IFSs.

The proper assessment of attribute weights plays an essential role in the MADM process because the variation of weight values may result in different final rankings of alternatives [18]. In general, the weights in MADM can be classified as subjective weights and objective weights depending on the method of information acquisition [26]. Subjective weights are obtained from preference information on attributes given by the DM, who provides subjective intuition or judgments on specific attributes. Objective weights are derived from the information in a decision matrix through mathematical models. The well-known approaches for generating subjective weights include AHP [28] and the Delphi method [17]. In terms of determining objective weights, one of the most-representative approaches is the entropy method, which expresses the relative intensities of attribute importance to signify the average intrinsic information transmitted to the DM [28,41]. Most research pertaining to MADM analysis in an IF environment has been executed using subjective weights [1,6,10]. However, there is little research on the extension of objective weighting approaches in MADM to IFSs.

In this study, we propose a new objective weighting method that combines intuitionistic fuzzy (IF) entropy measures to solve MADM problems. The entropy of a fuzzy set is a measure of the fuzziness of a fuzzy set. Although this is called entropy due to the concept's intrinsic similarity to Shannon entropy, the two functions measure fundamentally different types of uncertainty [31]. In the field of information theory, Shannon entropy is a measure of uncertainty associated with a random variable and is derived from the probability theory. However, the IF entropy measures used in our objective weighting method are non-probabilistic. De Luca and Termini [12] introduced a non-probabilistic entropy for fuzzy sets and formulated the axiomatic requirements with which an entropy measure should comply. Fuzziness is a feature of imperfect information and results from the lack of a clear distinction between the elements that do and do not belong to a set. Kaufmann [21] measured the degree of fuzziness of a fuzzy set using the metric distance between its membership function and the membership function of its nearest set. Szmidt and Kacprzyk [31] extended De Luca and Termini's axioms and proposed an entropy measure for IFSs called the IF entropy measure. Zeng and Li [42] expressed the axioms of Szmidt and Kacprzyk using the notation of interval-valued fuzzy sets. In summary, the concept of IF entropy that we apply in this paper differs from Shannon entropy; rather than drawing on information theory as is the case with Shannon entropy, our analysis is supported and evidenced by previous work on IF entropy [5,15,31,33,42].

The traditional entropy method focuses on the discrimination among data to determine attribute weights. If an attribute can discriminate the data more effectively, it is given a higher weight. In contrast, this study emphasizes the credibility of data in establishing attribute weights through IF entropy measures. Although it differs from the traditional entropy method, our method is nevertheless related to concepts associated with the traditional entropy method. Several IF entropy measures, including those introduced by Burillo and Bustince [5], Szmidt and Kacprzyk [31], Zeng and Li [42], Hung and Yang [15], and Vlachos and Sergiadis [32] as distinct theorems, were employed and compared using our objective weighting method. A computational experiment with simulation data was also designed to observe the ranking outcomes of attributes by applying different IF entropy measures.

The rest of this paper is organized as follows. Section 2 presents a collection of IF entropy measures and explains the determination of the specific measures used in the formal experiment. Section 3 illustrates an algorithm for the proposed method through a numerical example. Section 4 uses a computational experiment to compare some of the IF entropy measures implemented in the objective weighting method. The paper concludes in Section 5.

## 2. Entropy measures for intuitionistic fuzzy sets

### 2.1. Intuitionistic fuzzy sets

Atanassov [2] introduced the notion of intuitionistic fuzzy sets. An IFS $A$ in $X$ is defined as $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$. The values of $\mu_A(x)$ and $\nu_A(x)$ are the degrees of membership and non-membership of $x \in X$ in $A$, respectively. In addition, for each $x \in X$, $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$.

For each IFS $A$ in $X$, we call $\pi_A = 1 - \mu_A(x) - \nu_A(x)$ the intuitionistic index of $x$ in $A$. This is the hesitancy degree of $x$ to $A$. For every $A$, $B \in$ IFSs($X$), the following operations are defined:

1. $A \leqslant B$ if and only if $\mu_A(x) \leqslant \mu_B(x)$ and $\nu_A(x) \geqslant \nu_B(x)$ for all $x$ in $X$.
2. $A = B$ if and only if $A \leqslant B$ and $B \leqslant A$.
3. $A \cap B = \{x, \min(\mu_A(x), \mu_B(x)), \max(\mu_A(x), \mu_B(x)) | x \in X\}$.
4. $A \cup B = \{x, \max(\mu_A(x), \mu_B(x)), \min(\mu_A(x), \mu_B(x)) | x \in X\}$.

### 2.2. Interval-valued fuzzy sets

An interval-valued fuzzy set (IVFS) in $X$ is an expression $A$ given by $A = \{(x, M_A(x)) | x \in X\}$, where $M_A : X \to [0,1]$ defines the interval degree of membership of an element $x$ in the set $A \in X$ [29]. The interval $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ has a lower bound $M_{AL}(x)$ and an upper bound $M_{AU}(x)$ in the set $A$. Burillo and Bustince [5] and Sambuc [29] defined the following expressions for all $A$, $B \in$ IVFSs($X$):