Information Sciences 204 (2012) 70–81

Contents lists available at SciVerse ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Topological characterizations of covering for special covering-based upper approximation operators ☆

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ARTICLE INFO

Article history:
Received 22 June 2011
Received in revised form 19 January 2012
Accepted 1 April 2012
Available online 11 April 2012

Keywords:
Covering-based rough set
Unary covering
Closure operator
Base of a topology

ABSTRACT

The relationships among properties of covering approximation operators and their corresponding coverings have attracted intensive research in recent years. In particular, those among topological properties have drawn special attention because of their important applications in rough set theory. In this paper, we give topological characterizations of covering C for covering-based upper approximation operators FH, SH, TH and RH to be closure operators. We also give intuitive characterizations of covering C, and describe covering-based approximation space (U,C) as certain types of information exchange systems when SH or RH is a closure operator. By applying our new characterizations, we give inequalities about the relationship between the number of members in C and the number of elements in U, and discuss relationships among conditions for different covering based upper approximation operators to be closure operators. To the best of our knowledge, it is the first time that such characterizations, descriptions, inequalities and discussions are systematically considered in the literature of rough set theory. Furthermore, in this paper we also give several characterizations of unary coverings, an important type of coverings in studying relationships among basic concepts in covering-based rough sets, by the relationships among different types of covering-base approximation operators.

1. Introduction

Rough set theory has been acknowledged as a useful and powerful tool in data analysis particularly for dealing with granularity and vagueness [2,10,11,14,16,20–25,29,35–37,39,41–45,47,48,51,54,57]. The classical rough set theory is based on partitions of a universe [7,15,17,19,21,26,27,52,53,64], which imposes restrictions and limitations on many applications [56,67]. Zakowski generalized the classical rough set theory using coverings of a universe instead of partitions [56]. Such generalization leads to various covering approximation operators that are of both theoretical and practical importance [4,5,8,18,31,34,38,40,49,50,60,61,65]. In recent years, as data mining gets increasingly popular, the relationships between properties of covering-based approximation operators and their corresponding coverings have attracted intensive research [1,30,31,55,63,65–67,69,70]. It is worth noting that topological approaches have provided a valuable perspective and played an important role in rough set theory study [3,9,12,13,28,32,33,55,58,63]. Hence, topological properties have obtained a lot of attention [3,12,13,46,52,55,59,62,63]. In [66,68], Zhu and Wang discussed the relationship between properties of four types of covering-based upper approximation operators and their corresponding coverings. These four operators are named,

☆ This paper is supported by NSFC (Nos. 61070245, 10971185 and 11061004).
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http://dx.doi.org/10.1016/j.ins.2012.04.005

respectively, the first, the second, the third, and the fourth type of covering-based upper approximation operators, and are denoted by *FH*, *SH*, *TH* and *RH* respectively (definitions of these operators are provided in the next section). Specifically, Zhu and Wang gave characterizations of covering *C* for *FH*, *SH*, and *TH* to be closure operators by the following results.

Proposition 1.1 ([66,68]). *FH is a closure operator if and only if C satisfies one of the following conditions:*

(1) $\forall K_1, K_2 \in C$, $K_1 \cap K_2$ *is a union of elements of C;*
(2) *C is unary.*

Proposition 1.2 ([68,70]). *SH is a closure operator if and only if C satisfies the following condition:* $\forall x \in U$ *and* $K \in C$, *either* $K \subseteq Friends(x)$ *or* $K \cap Friends(x) = \phi$.

Proposition 1.3 ([68,70]). *TH is a closure operator if and only if C satisfies the following condition:* $\forall x \in U$ *and* $K \in C$, $K \cap CFriends(x)$ *is a union of elements of C.*

These results are important and interesting, but not complete. There are several important yet open problems following this direction.

First, to the best of our knowledge, so far there is no result on characterizations of covering *C* for *RH* to be a closure operator (only a necessary condition of covering *C* for *RH* to be a closure operator was presented in [66]). Hence, it is natural to ask:

Question 1.4. Can we find characterizations of covering *C* for *RH* to be a closure operator?

Notice that the relationship between definitions of *RH* and *FH* is similar to that between definitions of *SH* and *TH*; the latter two definitions (of *FH* and *TH*) just use *CFriends* instead of *Friends* as in the former ones (of *RH* and *SH*). It is important to know:

Question 1.5. If the answer for Question 1.4 is "yes", then what is the relationship among characterizations of covering *C* for the aforementioned four types of operators to be closure ones?

Second, the known characterizations of covering *C* that were presented in [66,68,70] for operators *FH*, *SH* and *TH* to be closure operators have nothing to do with topology, while being a closure operator is a notable topological property. It is then reasonable to consider:

Question 1.6. Do topological characterizations of covering *C* exist for *FH*, *SH*, *TH* and *RH* to be closure operators?

In this paper, we not only give answers to all these questions, but also give intuitive characterizations of covering *C*. By applying these characterizations, we describe covering-based approximation space (U,C) as certain types of information exchange systems when *SH* or *RH* is a closure operator, give two inequalities on the relationship between cardinalities of *C* and *U* when *SH* or *RH* is a closure operator, and discuss relationships among conditions for different covering based upper approximation operators to be closure ones. To the best of our knowledge, it is the first time that such characterizations, descriptions, inequalities and discussions are systematically considered in the literature of rough set theory.

Giving characterizations of unary coverings is useful, since unary coverings are important coverings in discussing relationships among basic concepts in covering-based rough sets [65,66,68,70]. In particular, a characterization of unary coverings was presented in [68] using the equality of two types of covering-based upper approximation operators: a covering *C* is a unary covering if and only if *TH* = *FH*. In this paper, we first give a topological characterization of unary coverings without using any covering-based upper approximation operators. As applications of this characterization, we give sufficient and necessary conditions from the perspective of topology for *FH* or *RH* to be a closure operator. Then we give several characterizations of unary coverings by the relationships among different types of covering-based approximation operators.

The rest of the paper is organized as follows. After giving fundamental concepts in Section 2, we present our main results in Sections 3–6. In these sections, we provide topological characterizations for the aforementioned four types of covering-based upper approximation operators to be closure operators, intuitive characterizations of covering *C*, descriptions of covering-based approximation space (U,C) using some special types of information exchange systems, and the inequalities about the relationship between the cardinalities of *C* and *U*. In Section 7, we discuss relationships among conditions for different covering based upper approximation operators to be closure operators. In Section 8, we give characterizations of unary covering by relationships among different types of covering-based approximation operators. This paper concludes in Section 9 with remarks and questions we shall consider in our future study.

2. Basic definitions

In this section, we present some basic concepts that are used in this paper. In the following discussion, unless it is mentioned specially, the universe of discourse *U* is considered finite. $P(U)$ denotes the family of all subsets of *U*. *C* is a family of subsets of *U*. If none of subsets in *C* is empty, and $\bigcup C = U$, then *C* is called a covering of *U*. We call ordered pair (U,C) a covering-based approximation space.