Information Sciences 180 (2010) 3914–3931

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Fuzzy transforms for compression and decompression of color videos

Ferdinando Di Martino [a,b], Vincenzo Loia [a], Salvatore Sessa [b,*]

[a] Università degli Studi di Salerno, Dipartimento di Matematica e Informatica, Via Ponte don Melillo, 84081 Fisciano, Italy
[b] Università degli Studi di Napoli Federico II, Dipartimento di Costruzioni e Metodi Matematici in Architettura, Via Monteoliveto 3, 80134 Napoli, Italy

A R T I C L E I N F O

Article history:
Received 28 March 2008
Received in revised form 2 June 2010
Accepted 21 June 2010

Keywords:
Fuzzy transform
Lukasiewicz t-norm
PSNR
JPEG

A B S T R A C T

We use fuzzy transforms for coding/decoding color frames of videos and we compare the results with the same frames reconstructed with the standard JPEG method. We classify all the frames in intra-frames and predictive frames by adopting a similarity measure based on the Lukasiewicz t-norm and a pre-processing phase which determines the best similarity threshold value. The compression is made on particular frames, called Δ-frames, obtained from a suitable difference defined on the values of the pixels of an intra-frame and a predictive frame. Under high compression rates, we see that the Peak Signal to Noise Ratio of the frames obtained with the fuzzy transforms is averagely close to the PSNR obtained with the JPEG method. We use the videos at URL sampl.eng.ohiostate.edu/~sampl/database.htm.

1. Introduction

We present a compression method of videos based on the application of the fuzzy transforms (for short, *F*-transforms). Our aim is to improve the video-compression process described in [21] and hence to obtain an optimal method with mean values of the Peak Signal to Noise Ratio (PSNR) very close to the PSNR calculated by using the standard JPEG method. Nowadays the digital video-compression is crucial in many fields like Internet Teleconferencing, High Definition TV (HDTV) and satellite communications. Then digital storage of videos is crucial, mainly when high compression rates are involved. A famous and consolidate standard method for video-compression is the Moving Picture Expert Group (MPEG) method [34] which adopts the Joint Photographic Experts Group (JPEG) standard technique [28] based on the use of the Discrete Cosine Transform (DCT) and the wavelets technology. Other known compression methods are:

- DV, a high-resolution digital video format used for video-cameras and camcorders that use DCT for lossy compression of the pixel data. The resulting video stream is transferred from the recording device via FireWire (IEEE 1394), a high-speed serial bus capable of transferring data up to 50 Mb/s;
- H261 and H2763, based on DCT and designed for two-way communication over ISDN lines (video conference) and supporting data rates which are multiples of 64 Kbit/s;
- DivX, a software application that uses the MPEG-4 standard in the compression of digital videos. It can be downloaded over a DSL/cable modem connection in a short time with no reduced visual quality.

The usage of fuzzy logic in image processing is a well known topic (e.g., [1,4,11–13,15,16,18,19,27,33,36–39]). In [7,8,30,32] the authors use the *F*-transforms in the compression/decompression of images and they prove that the PSNR is greater than that one obtained by using fuzzy relation equations with continuous triangular norms [5,6,14,17,20–26].

* Corresponding author. Tel.: +39 0812538907; fax: +39 0812538909.
E-mail addresses: fdimarti@unina.it (F.D. Martino), loia@unisa.it (V. Loia), sessa@unina.it (S. Sessa).

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doi:10.1016/j.ins.2010.06.030

In these last methods any monochromatic image is interpreted as a fuzzy matrix R whose entries are the normalized values of the pixels. The method based on fuzzy relation equations [10] concerns essentially the fact that the reconstructed image is the greatest or smallest solution (relation) of a system of fuzzy equations.

In accordance to the current literature [2,3,9,29–31,35], an F-transform in one variable is an operator which transforms a continuous function f defined on the interval $[a,b]$ in a n-dimensional vector by using an assigned family of fuzzy sets $A_1,\ldots,A_n$ which constitutes a fuzzy partition of $[a,b]$. Then one is able to define an inverse F-transform in order to convert the n-dimensional vector output in a continuous function which equals f up to an arbitrary quantity ε. We may limit this concept to the finite case by defining the discrete F-transform of a function f in one variable, even if it is not known a priori. An easy extension of this concept to functions in two variables allows to code/decode image processes (cfr. [7,8,30]).

Our proposal is to address the general idea (cfr. [5–8,20,21]) of coding/decoding the frames of a color video with F-transforms in the RGB space. We classify all the frames in intra-frames and predictive frames by adopting a similarity measure based on the Lukasiewicz t-norm and a pre-processing phase which determines the best similarity threshold value. The compression is realized via particular frames, called Δ-frames, obtained from a suitable difference defined on the values of the pixels of an intra-frame and a predictive frame. A frame is considered as a color image in each of the three bands, then it is divided in square submatrices called blocks, which in turn are coded with a suitable discrete F-transform. These blocks are successively decoded with the inverse discrete F-transform and recomposed by giving a new frame, to be compared with the original one via the PSNR. For sake of completeness, a comparison of both PSNR's obtained in our method and in the standard JPEG method (http://www.jj2000.epfl.ch) is presented.

This paper is organized as follows: in Section 2 we recall the main definitions and theorems from [30] concerning the concept of discrete (and its related inverse) fuzzy F-transform of a matrix with respect to an assigned fuzzy partition of a given interval $[a,b]$, in Section 3 we show how to apply the concepts of fuzzy transforms to compression/decompression of an image, in Section 4 we illustrate how to classify the frames of a video by using a similarity measure based on the Lukasiewicz t-norm and how to realize the compression process of these frames. Our tests involve 100 videos downloaded from the well known database at URL sampl.eng.ohio-state.edu/~sampl/database.htm. For brevity of presentation, we present in Section 5 only the results for three videos ("mom-daughter", "tennis" and "sflowg") and Section 6 concludes the paper.

2. Fuzzy transforms

We recall from [30] some essential definitions. Let $n \geqslant 3$ and $x_1, x_2,\ldots,x_n$ be points (nodes) of $[a,b]$ such that $x_1 = a < x_2 < \cdots < x_n = b$. We say that the fuzzy sets $A_1,\ldots,A_n$: $[a,b] \to [0,1]$ form a fuzzy partition of $[a,b]$ if.

(1) $A_i(x_i) = 1$ for every $i = 1,2,\ldots,n$;
(2) $A_i(x) = 0$ if x notin $(x_{i-1},xi+1)$, where $i = 1,2,\ldots,n$ and $x_0 = x_1 = a$, $x_{n+1} = x_n = b$;
(3) $A_i(x)$ is a continuous function on $[a,b]$;
(4) $A_i(x)$ is strictly increasing on the interval $[x_{i-1},x_i]$ for $i = 2,\ldots,n$ and is strictly decreasing on the interval $[x_i,x_{i+1}]$ for $i = 1,\ldots,n-1$;
(5) for every $x \in [a,b]$, $\sum_{i=1}^{n} A_i(x) = 1$.
The fuzzy sets $A_1,\ldots,A_n$ are called basic functions and we say that they constitute an uniform (or symmetric) fuzzy partition if the following additional properties hold:
(6) $x_i = a + h\cdot(i-1)$ for $i = 1, 2,\ldots,n$, where $h = (b-a)/(n-1)$;
(7) $A_i(x_i - x) = A_i(x_i + x)$ for every $x \in [0,h]$ and $i = 2,\ldots,n-1$;
(8) $A_{i+1}(x) = A_i(x-h)$ for every $x \in [x_i,x_{i+1}]$ and $i = 1, 2,\ldots,n-1$.

Property (6) assures that the nodes are equidistant. Our aim is to consider only the discrete case, i.e. functions f assuming assigned values in a finite set $P = \{p_1,\ldots,p_m\} \subseteq [a,b]$, f: $P \to [0,1]$. Moreover we need to suppose that the set P is sufficiently dense with respect to a fixed fuzzy partition $\{A_1,A_2,\ldots,A_n\}$ of $[a,b]$, i.e. if $m > n$ and for each $i = 1,\ldots,n$ there exists an index $j \in \{1,\ldots,m\}$ such that $A_i(p_j) > 0$. We say that the n-tuple $(F_1,\ldots,F_n)$ of real numbers, $F_i \in [0,1]$, is the discrete F-transform of f with respect to $\{A_1,A_2,\ldots,A_n\}$ if

$$F_i = \frac{\sum_{j=1}^{m} f(p_j)A_i(p_j)}{\sum_{j=1}^{m} A_i(p_j)} \tag{1}$$

for $i = 1,\ldots,n$. If all the Fi's are known, then the discrete inverse F-transform of f with respect to $\{A_1,A_2,\ldots,A_n\}$ is the function f_n^F: $P \to [0,1]$ defined as

$$f_n^F(p_j) = \sum_{i=1}^{n} F_i A_i(p_j) \tag{2}$$

for $j = 1,\ldots,m$. Then the following approximation theorem (cfr. [30, Theorem 5]) holds: