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On conditional diagnosability of the folded hypercubes ☆

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Abstract

The n-dimensional folded hypercube FQ_n, a variation of the hypercube proposed by Ahmed et al. [A. El-Amawy, S. Latifi, Properties and performance of folded hypercubes, IEEE Transactions on Parallel and Distributed Systems 2(3) (1991) 31–42], is an $(n+1)$-regular $(n+1)$-connected graph. Conditional diagnosability, a new measure of diagnosability introduced by Lai et al. [Pao-Lien Lai, Jimmy J.M. Tan, Chien-Ping Chuang, Lih-Hsing Hsu, Conditional diagnosability measures for large multiprocessor systems, IEEE Transactions on Computers 54(2) (2005) 165–175] can better measure the diagnosability of regular interconnection networks. This paper determines that under PMC-model the conditional diagnosability of FQ_n ($t_c(FQ_n)$) is $4n-3$ when $n=5$ or $n \geqslant 8$; $t_c(FQ_3)=3$, $t_c(FQ_4)=7$.

Keywords: PMC-model; Diagnosability; Conditional diagnosability

1. Introduction

In a multiprocessor system, some processors may fail. So processor fault identification plays an important role for reliable computing. The process of identifying faulty processors is called the diagnosis of the system. A system is said to be t-diagnosable if all faulty units can be identified provided the number of faulty units present does not exceed t. The *diagnosability* of a system is the maximal number of faulty processors that the system can guarantee to diagnose. The diagnosability of many interconnection networks have been explored [1–3,5–9,15,24,25].

For the purpose of self-diagnosis of a system, several different models have been proposed in the literature [20–22]. Preparata et al. [22] first introduced a model, the so-called PMC-model, for system level diagnosis in multiprocessor systems. In this model, it is assumed that a processor can test the faulty or fault-free status of another processor. Under the PMC-model, only processors with direct link are allowed to test each other. It is assumed that if a node is fault-free it should always give correct and reliable test results and if a node is faulty then its test result may be correct or incorrect.

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A collection of all test results is called a *syndrome*. For a given syndrome s, a fault set F of processors in the system is called to be *consistent* with the syndrome s, if the following two conditions are satisfied: for any test of a processor u by a processor v not in F.

(1) If v reports u to be faulty, then $u \in F$.
(2) If v reports u to be fault-free, then $u \notin F$.

Given a fault set F, since the test results of the processors in F may be correct or incorrect, there exist many syndromes which are consistent with F, we use $\sigma(F)$ to denote the set of all syndromes which are consistent with F. Two distinct sets F_1 and F_2 are said to be *indistinguishable*(resp. *distinguishable*) if $\sigma\ (F_1) \cap \sigma(F_2) \neq \emptyset$(resp. $\sigma(F_1) \cap \sigma(F_2) = \emptyset$).

Given a system, if all the neighbors of a processor v are faulty simultaneously, it is not possible to determine whether processor v is faulty or fault-free. So the diagnosability of a system is less than its minimum node degree. But in some large-scale multiprocessor systems like hypercubes or in some heterogenous environment, we can safely assume that all the neighbors of any node cannot fail at the same time [12]. Under this assumption, Esfahanian [12] introduced the restricted connectivity of multiprocessor system in 1989. And the restricted connectivity of many interconnection networks have been explored. Recently, Lai et al. [15] introduced the *conditional diagnosability* under this assumption and showed that the conditional diagnosability of the n-dimenisonal hypercube(Q_n) is $4n - 7$ when $n \geqslant 5$.

The hypercube is a very famous interconnection network. The folded hypercube is a variation of the hypercube by adding some edges. It was proposed by El-Amawy and Latifi [11]. An n-dimensional folded hypercube (FQ_n) has the same number of nodes as an n-dimensional hypercube and 2^{n-1} more edges than an n-dimensional hypercube. The diameter of an n-dimensional folded hypercube is reduced to about a half. At the same time the folded hypercube preserves the symmetric properties of the hypercube. Thus the folded hypercube is regarded as an attractive alternative to the hypercube and many of its properties have been explored [32,16,18,19,17,27,26].

In this paper, we show that the conditional diagnosability of FQ_n is $4n - 3$ when $n \geqslant 8$, which is about four times as large as that of the classical diagnosability of FQ_n.

The rest of this paper is organized as follows: In Section 2, we give some preliminaries; In Section 3, we prove the main result of this paper. Section 4 gives the conclusions and future directions of research.

2. Preliminaries and some lemmas

We follow [4] for terminologies and notations not defined here. Let G be a simple graph. For any vertex $v \in V(G)$, we use $N_G(v)$ to denote the neighboring vertex set of v in G. When G is clear from the context, we use $N(v)$ to replace $N_G(v)$ for simplification. We use $\delta(G)$ to denote the minimum degree of G. For any $S \subset V(G)$, we define $N_G(S) = \{v \in V(G) - S | \exists u \in S \text{such that} (u, v) \in E\}$. The connectivity of a graph G, denoted by $\kappa(G)$, is the minimum number of vertices whose deletion results in a disconnected graph or a trivial graph. Given a graph G, a super vertex cut of G is a vertex set $S \subset V(G)$ such that $G - S$ is disconnected and there is no isolated vertex in $G - S$.

The n-dimensional hypercube Q_n can be modeled as a graph $Q_n(V, E)$, and each node in V corresponds to an n-bit binary string. Let $\oplus$ denote the bitwise exclusive or (XOR) operation on binary numbers. For each node $u \in V$, let $a(u)$ and $\|a(u)\|$denote the binary address of the node u and the number of 1's in binary number $a(u)$, respectively. Two nodes u and v are adjacent in Q_n whenever $\|a(u) \oplus a(v)\| = 1$. It is easy to see that two vertices u, v in an n-dimension hypercube have common neighboring vertices if and only if $\|a(u) \oplus a(v)\| = 2$. And when they have common neighboring vertices, they have exactly two common neighboring vertices.

Folded hypercube is a variation of the hypercube obtained by adding some edges between the vertices of longest distance. So the vertex set of FQ_n is same to the vertex set of Q_n and the edge set of FQ_n is a superset of $E(Q_n)$. Two nodes u and v are adjacent in FQ_n whenever $\|a(u) \oplus a(v)\| = 1$ or n. An edge is called a complementary edge(resp. an ith edge) if it connects two nodes whose addresses differ in all bit positions(resp. the ith bit position). In the following we use nodes and their binary addresses interchangeably. Three-dimensional folded hypercube and hypercube are shown in Fig. 1.