Information Sciences 192 (2012) 244–254

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Stability analysis of swarms with interaction time delays

Qun Liu [a,*], Lanfen Wang [a], Xiaofeng Liao [b]

[a] College of Computer Science and Technology, ChongQing University of Posts and Telecommunications, 400065 ChongQing, PR China
[b] College of Computer Science and Technology, ChongQing University, 400030 ChongQing, PR China

ARTICLE INFO

Article history:
Available online 1 May 2011

Keywords:
Collective behavior
Time delays
Environment profiles
Aggregation
Swarms

ABSTRACT

This paper investigates the collective behavior of a general swarm model with communication time delays under different environment profiles, and the complex dynamic behavior of the delayed swarm model along a plane and a quadratic attractant/repellent profile are analyzed respectively too. It turns out that the swarm members can eventually converge to a finite region under certain conditions and the time delay plays an important role in the dynamic behavior of swarms. Finally numerical simulation results indicate that our theoretical analysis is correct.

1. Introduction

In recent years, researches on cooperative collective behavior and flocking control of multiple autonomous agents are becoming a focal subject. The basic problem in the researches is concerned with how to model and analyze the collective behavior in a group of entities due to their interactions, such as bird flocks, fish schools, ant swarms, multi-agents and multi-robots. The researches are important because swarms of autonomous agents can exhibit complex behavior which appears to transcend the individual ability of the agents, so the swarm intelligence can be used to solve the cooperation and consistency in many different distributed problems. That is to say, in order to achieve certain group goals, a group of relatively simple autonomous individuals can work together, such as foraging for foods or keeping formations. If we understand the collaborative and operational principles of such systems, the ideas for developing approaches of coordination and controlling of multi-robots, uninhabited autonomous vehicles, intelligent agents and intelligent traffic systems may be provided [9,21].

Biologists have been working on understanding and modeling of swarming behavior for a long time [2,13]. As we all know, the swarming behavior is a result of an interplay between a long range attraction and a short repulsion among the swarm members [2,19,20,23]. Many physicists have done important work on swarming behavior [1] too. The general approach they used is to model each individual as a particle and study the collective behavior through the swarm members' interactions [20,23]. The aim is to study the impact of the noise on the collective behavior. So many mathematical models have been proposed to address how the social group is organized in different environments. However, engineers have found the similarities between animal aggregation behaviors and coordination among multiple autonomous agents [7,24]. In recent years, engineering applications such as formation control of multi-robot teams and autonomous air vehicles emerged, and the interests of engineers on swarms increased too. In [7], the authors have described formation control strategies for autonomous air vehicles and multiple autonomous land vehicle teams. In [14,22], studies on cooperative control and coordination of a group of mobile robots to capture/enclose a target by making group formations have been made.

On the other hand, many researchers have done important work on swarm stability, oscillation and chaos. In [8], Beni and coworkers have considered a synchronous distributed control method for discrete one or two dimensional swarm structures. They have also proved stability in the presence of disturbances using Lyapunov methods. In [7], Gazi and Passino have

* Corresponding author. Tel./fax: +86 23 62461043.
E-mail addresses: liuqun@cqupt.edu.cn, liuqunsy@x263.net (Q. Liu).

0020-0255/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.ins.2011.04.040

presented a first-order continuous kinematic model for the swarm members. Then they have proposed a decentralized controller to analyze the swarm aggregation in n-dimensional space by using the idea of virtual forces. They have also showed that individuals can form a cohesive swarm in a finite time, and an explicit bound of the size of the cohesive swarm has been derived too. The results in [7] have been extended to a more general class of virtual force functions in [3]. In their later work [4–6], the same approach has been exploited to demonstrate the collective behavior of swarms moving in certain environments. In [18], authors have investigated the chaotic dynamic characteristics in swarm intelligence, but their swarm model was represented as an iterated function system (IFS).

However, due to the time delays existing in the individuals' communications, they often make system performance degraded and lead to the more complex dynamic behavior of swarm model, including divergence or instability, so it is very important to consider the time delays in swarm model. Swarm stability under time delays has been first considered in [15–17], in which these stability investigations have been limited only to either one or two dimensional space. In [11,12], the authors have researched the stability and oscillation of delay swarm model in n-dimensional space without considering the environmental impact.

In this paper, we propose a swarm model with communication time delays in certain environments. Its collective behavior and its stability have been studied by theoretical analysis. The results show that the delay swarm model in certain environment can display aggregating and cohesive dynamics under certain conditions. In order to prove our theoretical results, numerical simulations are given, well coinciding with the theoretical results, and more complex self-organized oscillatory motions are observed from the simulations too.

The rest of paper is organized as follows: in Section 2, we introduce the delayed swarm model at first. Section 3 is the analysis of aggregation and cohesion of the swarm dynamics under general condition, then the analysis of the behavior on the swarm along a plane and a quadratic attractant/repellent profile are also given. Some numerical simulations of collective oscillations on this swarm model are given in Section 4. Finally, Discussions and conclusions are given in Section 5.

2. Swarm model

A swarm of M agents in an n-dimensional Euclidean space is considered in this part. Each agent is modeled as a point mass. The motion of individual i is governed by the following delay differential equation.

$$\dot{x}^i(t) = -\nabla_{x^i}\sigma(x^i) + \sum_{j=1,j\neq i}^{M} g(x^i(t-\tau_{ij}(t)) - x^j(t-\varsigma_{ji}(t))), \quad i = 1,\ldots,M, \tag{1}$$

where $x^i \in R^n$ represents the position of individual $i, \sigma : R^n \to R$ represents the environment function which can be a profile of the attractant/repellent or the attractant/repellent substances (e.g., food/goal, or toxic chemicals/obstacle). For example, if $\sigma(y) < 0$, it represents the attractant substances or the goal; if $\sigma(y) = 0$, it represents a neutral; and if $\sigma(y) > 0$, it represents the repellent substances or an obstacle environment. The value of the term $-\nabla_{x^i}\sigma(x^i)$ represents the motion of the individuals toward regions due to the goals with higher nutrient concentration or away from regions because of the obstacle with high concentration of toxic substances, $g(\cdot)$ represents the function of mutual attraction or repulsion among the individuals, and it is also assumed to have the property of long range attraction and short range repulsion, $\tau_{ij}(t)$, $\varsigma_{ji}(t) \in [0,\tau]$ are communication time delays, where τ is a constant.

Since $g(\cdot)$ is the final result of mutual attraction and repulsion among the individuals, it can be expressed as an odd function as follows:

$$g(y) = -y[g_a(\|y\|) - g_r(\|y\|)], \quad y \in R^n, \tag{2}$$

where $g_a : R^+ \to R^+$ represents the magnitude of attraction term and has a long range, $g_r : R^+ \to R^+$ represents the magnitude of repulsion term and has a short range, $\|y\| = \sqrt{y^Ty}$ is the Euclidean norm.

We use the following attraction–repulsion function which satisfies the above Eq. (2) as in [7]:

$$g(y) = -y\left[a - b\exp\left(-\frac{\|y\|^2}{c}\right)\right], \tag{3}$$

where a, b are inter-individual attraction parameter and repulsion parameter respectively, a, b, c are positive constants, and $b > a$, then

$$g_a(\|y\|) = a, \quad g_r(\|y\|) = b\ \exp(-\|y\|^2/c).$$

As in [19], linear attraction and bounded repulsion are received as follows:

$$g_a(\|y\|) = a > 0, \quad g_r(\|y\|) \leqslant \frac{b}{\|y\|}. \tag{4}$$

In the following sections, we will first perform average motion and give the cohesion analysis for the swarm system (1) under conditions satisfied with some assumptions and profiles. Then we will analyze whether the individual with communication time delays can aggregate in a certain range during the evolution of the swarm.