Information Sciences 180 (2010) 2328–2339

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

The triangular pyramid: Routing and topological properties

S. Razavi [a], H. Sarbazi-Azad [b,*]

[a] School of Computer Science, Institute for Research in Fundamental Sciences (IPM), Tehran, Iran
[b] Department of Computer Engineering, Sharif University of Technology, Tehran, Iran

ARTICLE INFO

Article history:
Received 8 January 2007
Received in revised form 20 November 2009
Accepted 10 February 2010

Keywords:
Interconnection network
Triangular mesh
Triangular pyramid
Pyramid
Hamiltonian properties
Routing
Cycle embedding

ABSTRACT

In this paper, a new topology for multicomputer interconnection networks, based on triangular mesh, is proposed. The new network, referred to as the *triangular pyramid* (or *tripy* for short), has L levels of triangular mesh. We study some basic important properties of the proposed network as well as introduce a routing algorithm for the tripy network based on the routing of triangular meshes. We prove that this form of pyramidal network is Hamiltonian, Hamiltonian-connected, and pancyclic. We also prove that the proposed network is 6-colorable and conduct a brief comparison of the tripy and its traditional pyramid counterpart. Our results show that the proposed network has higher scalability, connectivity, and total network bandwidth while preserving the important properties of the traditional pyramid network.

1. Introduction

Designing parallel computers is a popular trend for gaining cost-effective computing. In these parallel computers, many processors are interconnected by an interconnection network where they cooperate to solve a large and complex problem [1,4,24]. Interconnection networks are currently being used for many different applications ranging from inter-IP connections in VLSI circuits to wide area computer networks [7]. An interconnection network can be modeled by a graph where a processor is represented by a node, and a communication channel between two processing nodes is represented by an edge between corresponding nodes [1,4,24]. Various topologies for interconnection networks have been proposed in the literature; these include *cubic* networks (e.g., meshes, tori, k-ary n-cubes, hypercubes, folded cubes, and hypermeshes), *hierarchical* networks (e.g., pyramids and trees), and *recursive* networks (e.g., RTCC networks, OTIS or swapped networks, WK-recursive networks, and star graphs) that have been widely studied in the literature for their topological properties [5,8–13,15,18,19,26,27], communication algorithms [3,7,10,11,13,19,21,22,26,29] and popular applications [4,24,14,16,32], embedding capabilities [6,9,23,25,28,29] and reliability and fault-tolerance issues [2,3,7,9,17,20,28,30,31].

Desirable properties of interconnection networks include [24] symmetry, small node degree d, diameter D, network cost (the product of the node degree and the network diameter, $d \times D$), high connectivity, scalability (both from a hardware and performance point of view), and fault-tolerance. The simplest physical connection pattern for the nodes is an n-node ring, R_n (or cycle C_n). In such a setting, each node has only two communication links, one to either of the two adjacent nodes. In an ideal setting, the direct connection of each node in an n-node network to every other node will allow one-hop communication between every pair of nodes. This pattern is known as a complete graph K_n. However, in this topology, the cost of actual implementation of the complete network grows excessively as the number of nodes increases. Most topologies introduced

* Corresponding author.
E-mail addresses: azad@sharif.edu, azad@ipm.ir (H. Sarbazi-Azad).

0020-0255/$ - see front matter
doi:10.1016/j.ins.2010.02.016

by researchers try to compromise between cost and performance, resulting in a wide range of different interconnection topologies, each with advantages and disadvantages [1,4,24].

A famous network topology that has been used as the base of both hardware architectures and software structures is the pyramid. By exploiting the inherent hierarchy at each level, pyramid structures can efficiently handle the communication requirements of various problems in graph theory, digital geometry, machine vision, and image processing [1,4,24]. In addition, the fault-tolerance properties of the pyramid [2] make it a promising network for reliable computing. Therefore, pyramids have gained much attention in the past [2,5,12,23,25].

The main problems with traditional pyramids are hardware scalability and poor network connectivity and bandwidth. To address these problems, we propose a new pyramidal network, *triangular pyramid*, which is based on triangular mesh instead of the traditional two-dimensional (2D) mesh employed by traditional pyramids. The new network preserves many desirable properties of traditional pyramid networks, but unlike them, here a node in the triangular pyramid may have more than one parent that can be exploited to effectively handle more communication patterns found in different parallel applications; higher connectivity can also help the tripy to simulate other network topologies more efficiently compared to its equivalent pyramid network.

The rest of the paper is organized as follows. In Section 2, the triangular mesh is defined, y and based on it, the new pyramidal network is introduced. We also introduce some useful notation and definitions that will be used in subsequent sections. In Section 3, some of the basic properties of the triangular pyramid such as the node degree, the diameter, and its addressing scheme are discussed. In Section 4, we prove that the tripy network is Hamiltonian, Hamiltonian-connected, and pancyclic. In Section 5, a brief comparison of the tripy and the pyramid network using basic network parameters is considered. Finally, Section 6 concludes this paper.

2. Triangular mesh and the triangular pyramid

In this section, we formally define the conventional pyramid, triangular mesh, and tripy networks and derive some topological properties of the tripy.

2.1. Definitions and basic properties

Definition 1. An $a \times b$ *mesh*, $M_{a,b}$, is a set of nodes $V(M_{a,b}) = \{(x,y)|1 \leqslant x \leqslant a,\ 1 \leqslant y \leqslant b\}$ where any two nodes (x_1,y_1) and (x_2,y_2) are connected by an edge iff $|x_1 - x_2| + |y_1 - y_2| = 1$ [25].

Definition 2. A *pyramid* of n levels, denoted by P_n, consists of a set of nodes $V(P_n) = \{(k,x,y)|0 \leqslant k \leqslant n,\ 1 \leqslant x,\ y \leqslant 2^k\}$. A node labeled $(k,x,y) \in V(P_n)$ is said to be a node at level k. All the nodes in level k form a $2^k \times 2^k$ mesh network. There are a total of $N = \sum_{k=0}^{n} 4^k = (4^{n+1} - 1)/3$ nodes in a P_n. In addition to the mesh connection between nodes at level $k \leqslant n$, each node (k,x,y) is also connected to nodes $(k+1,x-2,2y)$, $(k+1,2x,2y-1)$, $(k+1,2x-1,2y-1)$, and $(k+1,2x,2y)$, for $0 \leqslant k < n$ in level $k+1$, as *child* nodes. Also, the node is connected to node $(k-1, \lfloor \frac{x}{2} \rfloor, \lfloor \frac{y}{2} \rfloor)$, in level $k-1$ as its *parent* node [25].

Definition 3. A radix-n *triangular mesh* network, denoted as T_n, consists of a set of nodes $V(T_n) = \{(x,y)|0 \leqslant x + y < n\}$ where any two nodes (x_1,y_1) and (x_2,y_2) are connected by an edge if $|x_1 - x_2| + |y_1 - y_2| < n - 1$. The number of nodes and links in a T_n is equal to $n(n+1)/2$ and $3n(n-1)/2$, respectively. The degree of a node in this network can be 2, 3, or 6. The degree of node (x,y) is 2 when $x = y = 0$, or $x = 0$ and $y = n - 1$, or $x = n - 1$ and $y = 0$, and is 3 if $x = 0$ and $1 \leqslant y \leqslant n - 2$, or when $y = 0$ and $1 \leqslant x \leqslant n - 2$; otherwise, the node degree is 6. In Fig. 1, a T_5network is shown.

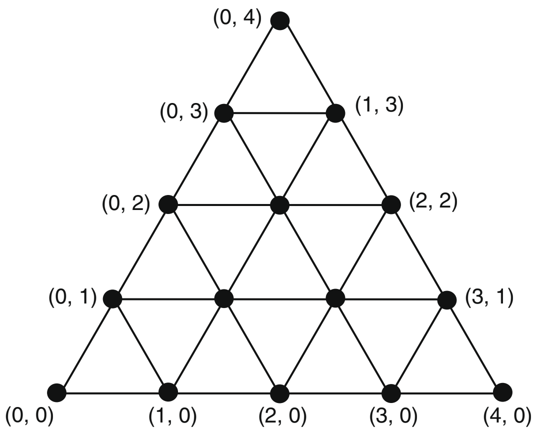

Fig. 1. The T_5.