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Choquet integrals of weighted intuitionistic fuzzy information

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ABSTRACT

The Choquet integral is a very useful way of measuring the expected utility of an uncertain event [G. Choquet, Theory of capacities, Annales de l'institut Fourier 5 (1953) 131–295]. In this paper, we use the Choquet integral to propose some intuitionistic fuzzy aggregation operators. The operators not only consider the importance of the elements or their ordered positions, but also can reflect the correlations among the elements or their ordered positions. It is worth pointing out that most of the existing intuitionistic fuzzy aggregation operators are special cases of our operators. Moreover, we propose the interval-valued intuitionistic fuzzy correlated averaging operator and the interval-valued intuitionistic fuzzy correlated geometric operator to aggregate interval-valued intuitionistic fuzzy information, and apply them to a practical decision-making problem involving the prioritization of information technology improvement projects.

1. Introduction

The concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov [1] to generalize the concept of Zadeh's fuzzy set [24]. Each element in an IFS is expressed by an ordered pair, and each ordered pair is characterized by a membership degree and a non-membership degree. The sum of the membership degree and the non-membership degree of each ordered pair is less than or equal to one. Since it was first introduced in 1986, IFS theory has been widely investigated and applied to a variety of fields [2,4,12,13,20,22]. Information aggregation is an interesting and important research topic in IFS theory that has been receiving more and more attention in recent years. Atanassov [1] defined some basic operations and relations of IFSs, including "intersection", "union", "complement", "addition" and "multiplication". Atanassov and Gargov [3] extended the results of Atanassov [1] to interval-valued IFSs. De et al. [6] further defined the concentration, dilation and normalization of IFSs, and proved some propositions in this context. Deschrijver and Kerre [9] introduced some aggregation operators on the lattice L^*, and defined the special classes of binary aggregation operators, based on t-norms on the unit interval (Deschrijver and Kerre [7,8,10] showed that IFSs could be seen as L-fuzzy sets). Xu and Yager [23] developed some geometric aggregation operators and applied them to multiple attribute decision making based on IFSs. Xu [19] developed the intuitionistic fuzzy weighted averaging operator, the intuitionistic fuzzy ordered weighted averaging operator and the intuitionistic fuzzy hybrid aggregation operator, for aggregating intuitionistic fuzzy values, and studied their various properties. All of the existing intuitionistic fuzzy aggregation operators only consider situations where all the elements in an IFS are independent, i.e., they only consider the addition of the importance of individual elements. However, in many practical situations, the elements in an IFS are usually correlative; for example, Grabisch [11] and Torra [16] gave the following classical example: "We are to evaluate a set of students in relation to three subjects: {mathematics, physics, literature}, we want to give more

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importance to science-related subjects than to literature, but on the other hand we want to give some advantage to students that are good both in literature and in any of the science-related subjects". Therefore, we need to find some new ways to deal with these situations in which the decision data in question are correlative. The Choquet integral [5] is a very useful way of measuring the expected utility of an uncertain event, and can be used to depict the correlations of the data under consideration.

Motivated by the correlation properties of the Choquet integral, in this paper we propose some intuitionistic fuzzy aggregation operators, whose prominent characteristic is that they can not only consider the importance of the elements or their ordered positions, but also reflect the correlations of the elements or their ordered positions. Then we extend the proposed operators to the interval-valued intuitionistic fuzzy environment.

2. Intuitionistic fuzzy aggregation operators based on the Choquet integral

Zadeh's fuzzy sets [24–26] are a very popular tool to describe imprecise or vague information in a variety of applied fields [4] such as management, engineering, artificial intelligence, data mining, pattern recognition and soft computing. The fundamental characteristic of a fuzzy set is that it assigns to each of its elements a membership degree, and a non-membership degree equal to one minus the membership degree. Atanassov [1] extended Zadeh's fuzzy sets to intuitionistic fuzzy sets (IFSs), which assign to each of their elements a membership degree and a non-membership degree under the constraint that the sum of the two degrees does not exceed one.

Given a fixed set $X = \{x_1, x_2, \ldots, x_n\}$, an IFS is defined as [1]

$$A = \{\langle x_i, t_A(x_i), f_A(x_i)\rangle | x_i \in X\}, \tag{1}$$

which assigns to each element x_i a membership degree $t_A(x_i)$ and a non-membership degree $f_A(x_i)$ under the condition

$$0 \leqslant t_A(x_i) + f_A(x_i) \leqslant 1, \quad \text{for all } x_i \in X. \tag{2}$$

In [18,23], we called the ordered pair $\alpha(x_i) = (t_\alpha(x_i), f_\alpha(x_i))$ an intuitionistic fuzzy value (IFV), where

$$t_\alpha(x_i) \in [0,1], \quad f_\alpha(x_i) \in [0,1], \quad t_\alpha(x_i) + f_\alpha(x_i) \leqslant 1, \tag{3}$$

and gave some useful operations on IFVs, as follows:

Let $\alpha(x_i) = (t_\alpha(x_i), f_\alpha(x_i))$ and $\alpha(x_j) = (t_\alpha(x_j), f_\alpha(x_j))$ be two IFVs; then

(1) $\alpha(x_i) \oplus \alpha(x_j) = (t_\alpha(x_i) + t_\alpha(x_j) - t_\alpha(x_i)t_\alpha(x_j), f_\alpha(x_i)f_\alpha(x_j))$;
(2) $\alpha(x_i) \otimes \alpha(x_j) = (t_\alpha(x_i)t_\alpha(x_j), f_\alpha(x_i) + f_\alpha(x_j) - f_\alpha(x_i)f_\alpha(x_j))$;
(3) $\lambda\alpha(x_i) = (1 - (1 - t_\alpha(x_i))^\lambda, (f_\alpha(x_i))^\lambda), \ \lambda > 0$;
(4) $(\alpha(x_i))^\lambda = ((t_\alpha(x_i))^\lambda, 1 - (1 - f_\alpha(x_i))^\lambda), \ \lambda > 0$,
where the results are also IFVs and have the following properties:
(1) $\alpha(x_i) \oplus \alpha(x_j) = \alpha(x_j) \oplus \alpha(x_i)$;
(2) $\alpha(x_i) \otimes \alpha(x_j) = \alpha(x_j) \otimes \alpha(x_i)$;
(3) $\lambda(\alpha(x_i) \oplus \alpha(x_j)) = \lambda\alpha(x_i) \oplus \lambda\alpha(x_j)$, $\lambda > 0$;
(4) $(\alpha(x_i) \otimes \alpha(x_j))^\lambda = (\alpha(x_i))^\lambda \otimes (\alpha(x_j))^\lambda$, $\lambda > 0$;
(5) $(\lambda_1 + \lambda_2)\alpha(x_i) = \lambda_1\alpha(x_i) \oplus \lambda_2\alpha(x_i)$, $\lambda_1, \lambda_2 > 0$;
(6) $(\alpha(x_i))^{\lambda_1+\lambda_2} = (\alpha(x_i))^{\lambda_1} \otimes (\alpha(x_i))^{\lambda_2}, \ \lambda_1, \lambda_2 > 0$.

Any two IFVs can be compared by the following method [18,22]:

Let $\alpha(x_i) = (t_\alpha(x_i), f_\alpha(x_i))$ and $\alpha(x_j) = (t_\alpha(x_j), f_\alpha(x_j))$ be two IFVs, $s(\alpha(x_i)) = t_\alpha(x_i) - f_\alpha(x_i)$ and $s(\alpha(x_j)) = t_\alpha(x_j) - f_\alpha(x_j)$ be the scores of $\alpha(x_i)$ and $\alpha(x_j)$, respectively, and let $h(\alpha(x_i)) = t_\alpha(x_i) + f_\alpha(x_i)$ and $h(\alpha(x_j)) = t_\alpha(x_j) + f_\alpha(x_j)$ be the accuracy degrees of $\alpha(x_i)$ and $\alpha(x_j)$, respectively; then

- If $s(\alpha(x_i)) < s(\alpha(x_j))$, then $\alpha(x_i)$ is smaller than $\alpha(x_j)$, denoted by $\alpha(x_i) < \alpha(x_j)$;
- If $s(\alpha(x_i)) = s(\alpha(x_j))$, then
  (1) If $h(\alpha(x_i)) = h(\alpha(x_j))$, then $\alpha(x_i)$ and $\alpha(x_j)$ represent the same information, i.e., $t_\alpha(x_i) = t_\alpha(x_j)$ and $f_\alpha(x_i) = f_\alpha(x_j)$, denoted by $\alpha(x_i) = \alpha(x_j)$;
  (2) If $h(\alpha(x_i)) < h(\alpha(x_j))$, then $\alpha(x_i)$ is smaller than $\alpha(x_j)$, denoted by $\alpha(x_i) < \alpha(x_j)$.

In the above method, the relation between the score function s and the accuracy function h is similar to the relation between the mean and variance in statistics. It is well known that an efficient estimator is a measure of the variance of an estimate's sampling distribution in statistics; the smaller the variance, the better the performance of the estimator, and thus it is reasonable and appropriate to stipulate that the higher the score and the accuracy degree of an IFV, the greater the IFV.