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Discrete time credibilistic processes: Construction and convergences

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ABSTRACT

In this paper, we discuss a method of constructing a credibilistic process which is a family of fuzzy variables on credibility space. Applying the extension theorem in credibility theory, the finite or infinite horizon credibilistic process is made up from a family of credibilistic kernels. Also, for the Markov case, convergence theorems are given and credibilistic risk models for reward processes are considered, whose risk is completed by the recursive equation.

1. Introduction

The credibility theory, originated by Liu [7,8], is a mathematical fuzzy model for studying and describing the phenomena of uncertainty, which is on the basis of the axiomatization of credibility measures and provides a powerful tool for analysis of the real uncertain problem (e.g. [3–5]).

Recently, Liu [9] has presented the concept of fuzzy process, called credibilistic process here, which is a family of fuzzy variables on credibility space, for which the much wider application to the area of uncertain sequential decision making (cf. [2]) can be imaged from judging the wide application of credibility theory. For the modellization of a real problem by using of fuzzy process, it is more useful to establish a method of constructing it.

In this paper, applying the credibility extension theorem [10], we will give a construction theorem by which a credibilistic process is constituted from a family of credibilistic kernels. For an infinite horizon credibilistic process constructed by Markov kernels, some convergence theorems are given. Also, credibilistic risk models for reward processes is being considered, whose risk is computed by the corresponding recursive equations. For the approach by possibility theory, for example, Avrachenkov [1], Thomason [13] and Kurano et al. [6]. For recent developments for fuzzy theory and fuzzy logic, refer to [16,17]. In Section 2, we recall such basic concept of credibility theory as a credibility measure, fuzzy variables. Reviewing the credibility extension theorem [10], we give the fundamental lemma which plays a key role in establishing a finite or infinite horizon credibilistic process. In Section 3, the construction theorem is given. In Section 4, a convergence theorem is developed for an infinite horizon credibilistic process. Also, credibilistic risk model is considered.

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2. Credibilistic process and basic lemmas

In this section, we shall give some definitions and basic lemmas on credibility space referring to [7,8,10] and the problem to be considered in sequel is specified.

For any non-empty set X, we denote by $\mathscr{P}(X)$ the power set of X. Let Θ be an arbitrary non-empty set. If the following four axioms are satisfied, the set function Cr is said to be a credibility measure on $\mathscr{P}(\Theta)$.

Axiom 1. $\mathrm{Cr}\{\Theta\} = 1$.

Axiom 2. Cr is increasing, i.e., $\mathrm{Cr}\{A\} \leqslant \mathrm{Cr}\{B\}$ whenever $A \subset B$.

Axiom 3. Cr is self-dual, i.e., $\mathrm{Cr}\{A\} + \mathrm{Cr}\{A^c\} = 1$ for any $A \in \mathscr{P}(\Theta)$.

Axiom 4. $\mathrm{Cr}\{\cup_i A_i\} = \sup_i \mathrm{Cr}\{A_i\}$ for any $\{A_i\} \subset \mathscr{P}(\Theta)$ with $\sup_i \mathrm{Cr}\{A_i\} < 0.5$.

Then the triplet $(\Theta, \mathscr{P}(\Theta), \mathrm{Cr}\{\cdot\})$ is called a credibility space.

Let $\Re$ be the set of real numbers. For any non-empty set X, a function $p : X \to [0,1]$ is said to satisfy Condition $\mathscr{K}$ with X if the following (1a) and (1b) hold:

$$\sup_{x \in X} p(x) \geqslant 0.5, \tag{1a}$$

$$p(x^*) + \sup_{x \neq x^*} p(x) = 1 \;\; \mathit{if} \;\; p(x^*) \geqslant 0.5. \tag{1b}$$

The following extension lemma given by Li and Liu [10] shows that any function satisfying Condition $\mathscr{K}$ induces a credibility measure on $\mathscr{P}(X)$.

Lemma 1 (Credibility extension theorem [10]). *For any non-empty set X, let p be any function satisfying Condition $\mathscr{K}$ with X. Then, a set function $\mathrm{Cr}\{\cdot\}$ defined by the following (2) and (3) becomes a credibility measure on $\mathscr{P}(X)$:*

$$\mathrm{Cr}\{A\} = \begin{cases} p(A) & \mathit{if} \;\; p(A) < 0.5, \\ 1 - p(A^c) & \mathit{if} \;\; p(A) \geqslant 0.5, \end{cases} \quad (A \in \mathscr{P}(X)) \tag{2}$$

where for $D \in \mathscr{P}(X)$

$$p(D) = \sup_{x \in D} p(x). \tag{3}$$

A function from a credibility space $(\Theta, \mathscr{P}(\Theta), \mathrm{Cr}\{\cdot\})$ to $\Re$ is called a fuzzy variable. A function $p : \Re \to [0,1]$ is called a prescriptive function of a fuzzy variable ξ if it satisfies that for any $A \in \mathscr{P}(\Re)$,

$$\mathrm{Cr}\{\xi \in A\} = \begin{cases} p(A) & \mathit{if} \;\; p(A) < 0.5, \\ 1 - p(A^c) & \mathit{if} \;\; p(A) \geqslant 0.5, \end{cases} \tag{4}$$

where $p(D)$ is defined by (3) $(D \in \mathscr{P}(\Re))$. A family of fuzzy variables is called a credibilistic process. This paper treats with the discrete time credibilistic process which is a sequence of fuzzy variables, say $\{\xi_t : t = 0, 1, \ldots\}$.

In the succeeding sections, we will show how to construct a credibilistic process and the corresponding prescriptive functions. To the end, we need Lemma 2 below.

For any non-empty set X, the set of all functions $p : X \to [0, 0.5]$ satisfying Condition $\mathscr{K}$ with X will be denoted by $\mathscr{K}(X)$ emphasizing the domain X. For any non-empty sets X and Y, a credibilistic kernel on Y given X is a function $q(\cdot|\cdot)$ on $Y \times X$ which satisfies that $q(\cdot|x) \in \mathscr{K}(Y)$ for all $x \in X$. The set of all credibilistic kernels on Y given X will be denoted by $\mathscr{K}(Y|X)$. The following lemma plays a major role in establishing a finite horizon credibilistic process.

Lemma 2. *Let X, Y be any non-empty sets. Let $q \in \mathscr{K}(Y|X)$. For any $p \in \mathscr{K}(X)$, we define a function $g : X \times Y \to [0, 0.5]$ by*

$$g(x, y) = p(x) \wedge q(y|x) \; \mathit{for} \; x \in X, \; y \in Y. \tag{5}$$

Then g satisfies Condition $\mathscr{K}$ with $X \times Y$, that is, $g \in \mathscr{K}(X \times Y)$.

Proof. We prove that $g(x,y)$ satisfies Condition $\mathscr{K}$ with $X \times Y$. Noting that $\sup_x p(x) = 0.5, \sup_y q(y|x) = 0.5$, we have

$$\sup_{(x,y) \in X \times Y} g(x,y) = \sup_{x \in X, y \in Y} p(x) \wedge q(y|x) = \sup_x \left(\sup_y p(x) \wedge q(y|x) \right) = \sup_x p(x) \wedge \sup_y q(y|x) = \sup_x p(x) \wedge 0.5 = 0.5 \wedge 0.5 = 0.5.$$

The second equality follows easily while the third follows from that $\sup_{y \in Y} a \wedge h(y) = a \wedge \sup_{y \in Y} h(y)$ for any constant a and function h on Y. If $g(x^*, y^*) = 0.5$, then it follows from (5) that $p(x^*) = 0.5, q(y^*|x^*) = 0.5$.

$$\sup_{(x,y) \neq (x^*,y^*)} g(x,y) = \left(\sup_{y \neq y^*} p(x^*) \wedge q(y|x^*) \right) \vee \left(\sup_{\substack{y \\ x \neq x^*}} p(x) \wedge q(y|x) \right) = (0.5 \wedge (1 - q(y^*|x^*))) \vee ((1 - p(x^*)) \wedge 0.5) = 0.5. \tag{6}$$