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Optimal consumption and portfolio choice with ambiguity and anticipation ☆

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Abstract

This paper, adopting the recursive multiple-priors utility, studies the optimal consumption and portfolio choice in a Merton-style model with anticipation when there is a difference between ambiguity and risk. The fundamental issue is what the effects of ambiguity and anticipation on the investor's behavior are. In the case of a logarithmic felicity function, the paper also shows that no hedging demand arises that is affected by both ambiguity and anticipation. Finally, the optimal portfolio is derived in terms of Malliavin derivatives and stochastic integrals.

Keywords: Ambiguity; Recursive multiple-priors utility; Anticipation; Optimal consumption and portfolio; Enlargement of filtrations; Malliavin derivatives

1. Introduction

In economic analysis, it is intuitive that many choice situations feature 'Knightian uncertainty' or 'ambiguity' and that these are different from 'risk'. The Ellsberg Paradox [9] tells us that this difference is behaviorally significant, which suggests that there are two dimensions of the decision-maker's beliefs about the likelihoods of events: risk and ambiguity. In standard models, ambiguity is neglected or it is assumed that the decision-maker is indifferent to it.

Another motivation for the analysis to follow comes from the finance literature on anticipation. In the real world, sometimes an investor makes consumption and investment decisions based on so-called 'insider trading' or 'inside-information' (see [1,12–14,20,28]). More precisely, it is supposed that the investor possesses some additional information possibly distorted by 'noise' from the very beginning.

To incorporate ambiguity, this paper asks the following question: How does a decision-maker choose when he is averse to ambiguity and capable of getting 'inside-information'? The first step in dealing with this question is to formulate a utility function that permits the difference between risk and ambiguity that is indicative

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of anticipation. Chen and Epstein [3] provide such a difference without the above anticipation. They call their model recursive multiple-priors utility. This paper adapts their formulation to an anticipative environment.

The issues are as follows: (i) How is the anticipation coped with. (ii) What are the effects of ambiguity and anticipation on the investor's behavior?

To solve these questions, we show that the separation principle (e.g., [15]) still holds for recursive multiple-priors utility. In particular, optimality is consistent with the use of any measure in the set of priors.

The characterization of optimal consumption and portfolio choice with anticipation and without ambiguity can be found in Fei and Wu [12,13] and Fei et al. [14]. There is voluminous literature studying the consumption and portfolio choice in the expected utility framework (see [10–13,21,22,28,17,25,29], etc.). Chen and Epstein [3] formulate the recursive multiple-priors utility in continuous time. They also apply this utility to a Lucas-style representative agent model in order to study asset pricing implications.

Concerning the characterization of optimal consumption and portfolio choice with both anticipation and ambiguity, we find that ordinary martingale methods and Malliavin calculus can be used to solve the agent problem.

Finally, we provide special cases of logarithmic and power felicity functions that deliver closed form solutions. In the logarithmic case, the effect of ambiguity is that the agent myopically holds a mean-variance efficient portfolio but with distorted mean values of asset returns. In spite of anticipation and ambiguity, there is no hedging demand. Both ambiguity and anticipation affect optimal portfolio.

This paper is organized as follows. Section 2 defines recursive multiple-priors utility with anticipation. In Section 3, we apply this utility to studying optimal consumption and portfolio choice in a Merton-style model with anticipation. Two special cases are provided in Section 4. Finally, we conclude in Section 5, mentioning possible further research topics.

2. Recursive multiple-priors utility

This section adopts those in Chen and Epstein [3] and defines recursive multiple-priors utility with anticipation.

2.1. Terminal information distorted by noise

Let time be continuous in the finite horizon $[0,T]$. There is a complete filtered probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}, P)$ on which a d-dimensional standard Brownian motion on $\mathbf{R}^d$, $W = (W^1, \ldots, W^d)^{\mathrm{T}}$, is defined.

The decision-maker's available inside information is considered. More precisely, suppose that the investor possesses from the very beginning some additional information about the value of a certain $\mathscr{F}$-measurable random variable L. Let

$$L \triangleq \{L_i\}_{i=1}^d = \left\{\lambda_i W_T^i + (1-\lambda_i)\varepsilon_i\right\}_{i=1}^d,$$

where ε_i are mutually independent standard normal variables, independent of W and $\lambda_i \in [0,1]$. The investor can choose portfolio and consumption from the class of $\{\mathscr{G}_t\}$-adapted processes, where $\{\mathscr{G}_t\}_{0\leqslant t\leqslant T}$ is given by

$$\mathscr{G}_t = \mathscr{F}_t \vee \sigma(L), \quad 0 \leqslant t \leqslant T.$$

We assume that the dynamics of $d+1$ asset prices are described by the following stochastic differential equations:

$$\mathrm{d}S_t^0 = S_t^0 r_t \mathrm{d}t, \quad S_0^0 = 1,$$
$$\mathrm{d}S_t^i = S_t^i \left[\mu_t^i \mathrm{d}t + \sum_{j=1}^d \sigma_t^{ij} \mathrm{d}W_t^j\right], \quad S_0^i = s^i, \ \ i = 1, \ldots, d,$$

where S_t^0 is the price of a bond and S_t^i is the price of the ith stock at time $t \in [0,T]$.

For the above random vector L, we shall provide its characterization by using the technique of Fei and Wu [12] on 'enlargement of filtration'. Along the lines, we have the following theorem.