Information Sciences 179 (2009) 4057–4069

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Fuzzy random renewal reward process and its applications

Shuming Wang *, Junzo Watada [1]

Graduate School of Information, Production and Systems, Waseda University, 2-7 Hibikino, Wakamatsu, Kitakyushu 808-0135, Fukuoka, Japan

ARTICLE INFO

Article history:
Received 8 April 2008
Received in revised form 6 August 2009
Accepted 8 August 2009

Keywords:
Renewal process
Renewal reward theorem
Fuzzy random variable
Archimedean t-norm
$\top$-Independence

ABSTRACT

This paper studies a renewal reward process with fuzzy random interarrival times and rewards under the $\top$-independence associated with any continuous Archimedean t-norm $\top$. The interarrival times and rewards of the renewal reward process are assumed to be positive fuzzy random variables whose fuzzy realizations are $\top$-independent fuzzy variables. Under these conditions, some limit theorems in mean chance measure are derived for fuzzy random renewal rewards. In the sequel, a fuzzy random renewal reward theorem is proved for the long-run expected reward per unit time of the renewal reward process. The renewal reward theorem obtained in this paper can degenerate to that of stochastic renewal theory. Finally, some application examples are provided to illustrate the utility of the result.

© 2009 Elsevier Inc. All rights reserved.

1. Introduction

Renewal reward processes are an important sort of renewal models and have a wide range of real-life applications (see [9,20]). Stochastic renewal theory, based on probability theory, is well developed [1,2,8,31,32]. It is well-known that in stochastic renewal reward processes, the interarrival times and rewards are assumed to be independent and identically distributed (i.i.d.) random variables, and the stochastic renewal reward theorem is one of the most significant results in this area. On the other hand, in order to deal with vague or fuzzy uncertainty in renewal processes, several researchers recently investigated fuzzy renewal processes in which the interarrival times and rewards are assumed to be imprecise and are characterized by fuzzy variables. For example, Zhao and Liu [41] discussed a fuzzy renewal process generated by a sequence of i.i.d. positive fuzzy variables and obtained a fuzzy elementary renewal theorem and a fuzzy renewal reward theorem, respectively. Hong [11] discussed a renewal process in which interarrival times and rewards are depicted by L—R fuzzy numbers under t-norm-based fuzzy operations.

In practical applications, randomness and fuzziness often coexist in a single process and thus are required to be considered simultaneously. In such cases, uncertainty cannot be handled in a satisfactory manner by using only either random variables or fuzzy numbers; we therefore need to combine the two and turn to a new tool to deal with this twofold uncertain process. The fuzzy random variable was introduced by Kwakernaak [15,16] in 1978 to study randomness and fuzziness at the same time, and it was defined as a function from a probability space to a collection of fuzzy numbers with certain measurability requirements. Later on, some variants and extensions were developed by other researchers for different purposes; see for example, Kruse and Meyer [14], Liu and Liu [21], López-Diaz and Gil [25], Luhandjula [26], and Puri and Ralescu [29]. Based on the concept of the fuzzy random variable, some renewal processes in fuzzy random environments have been discussed in the literature. For instance, Hwang [12] investigated a renewal process in which the interarrival times are assumed

* Corresponding author. Mobile: +81 80 3228 1030; fax: +81 93 692 5179.
E-mail addresses: smwangips@gmail.com (S. Wang), junzow@osb.att.ne.jp (J. Watada).
[1] Mobile: +81 90 3464 4929; fax: +81 93 692 5179.

0020-0255/$ - see front matter © 2009 Elsevier Inc. All rights reserved.
doi:10.1016/j.ins.2009.08.016

as i.i.d. fuzzy random variables, and proved an almost sure convergence theorem with the probability measure for the renewal rate. Modeling the rewards as i.i.d. fuzzy random variables, Popova and Wu [30] studied a fuzzy random renewal reward process and derived a theorem for the long-run average reward in the form of a level-wise convergence with the probability of one. Zhao and Tang [42] derived some other properties of fuzzy random renewal processes, and obtained a Blackwell's renewal theorem and a Smith's key renewal theorem for fuzzy random interarrival times. Furthermore, Li et al. [17] introduced the fuzzy random variable into delayed renewal processes, and discussed a fuzzy random delayed renewal process as well as a fuzzy random equilibrium renewal process which is a special case of the former.

In all studies on fuzzy random renewal processes mentioned above, the operations of fuzzy realizations of fuzzy random variables which are fuzzy numbers or fuzzy variables, are based on an independence associated with the minimum t-norm (Min-independence), or the extension principle with minimum t-norm. Nevertheless, a number of practical applications in the past two decades show that the classical extension principle (or Min-independence) is not always the optimal way to combine fuzzy numbers (for example, in image processing [4,6], measurement theory [34,37], fuzzy control [3,28], type-2 fuzzy system [19,27], and artificial neural networks [24]). The operations associated with different kinds of t-norms may be required for fuzzy numbers in different specific situations and applications. A more general extension principle makes use of a general t-norm operator. Such a generalized extension principle yields different operations for fuzzy numbers or fuzzy variables, in accordance with different t-norms. Recently, several studies have been reported which focus on such t-norm-based operations of fuzzy numbers (see [5,7,10,13]), and fuzzy random variables (see [34,35,38,39]). In particular, Wang et al. [39] have discussed a fuzzy random renewal process under the t-norm-based extension principle, and proved a fuzzy random elementary renewal theorem for the long-run expected renewal rate.

As a continuation of the work [39], in this paper we discuss a renewal reward process with fuzzy random interarrival times and rewards under the independence with t-norms ($\top$-independence), which induces the (generalized) t-norm-based extension principle for the operations of fuzzy realizations of fuzzy random variables; and we derive a new fuzzy random renewal reward theorem for the long-run expected average reward. In contrast with the fuzzy random renewal processes in [12,17,30,42] whose results hold only for the minimum t-norm, the renewal reward theorem obtained in this paper can be applied to more general situations using the class of continuous Archimedean t-norms, such as the product t-norm, Dombi t-norm, and Yager t-norm. Additionally, the results obtained in this paper well degenerates to the classical renewal reward theorem in the stochastic process.

The remainder of this paper is organized as follows. In Section 2, we recall some preliminaries on $\top$-independent fuzzy variables and fuzzy random variables. Section 3 discusses a fuzzy random renewal reward process, and derives a fuzzy random renewal reward theorem. In Section 4, we provide two applications to further explain how to use the fuzzy random renewal reward theorem obtained above. Finally, our conclusions are drawn in Section 5.

2. Preliminaries

2.1. $\top$-independent fuzzy variables

Given a universe Γ, let Pos be a set function defined on the power set $\mathscr{P}(\Gamma)$ of Γ. The set function Pos is said to be a possibility measure if it satisfies the following conditions

(**Pos1**) $\mathrm{Pos}(\emptyset) = 0$, and $\mathrm{Pos}(\Gamma) = 1$;
(**Pos2**) $\mathrm{Pos}(\bigcup_{i\in I} A_i) = \sup_{i\in I}\mathrm{Pos}(A_i)$ for any subclass $\{A_i | i \in I\}$ of $\mathscr{P}(\Gamma)$, where I is an arbitrary index set.

The triplet $(\Gamma, \mathscr{P}(\Gamma), \mathrm{Pos})$ is called a *possibility space*. Based on possibility measure, a self-dual set function Cr, named *credibility measure* (see [18]), is defined as

$$\mathrm{Cr}(A) = \frac{1}{2}(1 + \mathrm{Pos}(A) - \mathrm{Pos}(A^c)), \quad A \in \mathscr{P}(\Gamma), \tag{1}$$

where A^c is the complement of A.

Let $\Re$ be the set of real numbers. A function $Y : \Gamma \to \Re$ is said to be a fuzzy variable defined on Γ, and the possibility distribution μ_Y of Y is defined by $\mu_Y(t) = \mathrm{Pos}\{Y = t\},\ t \in \Re$, which is the possibility of event $\{Y = t\}$. A fuzzy variable Y is said to be positive almost surely, if $\mathrm{Cr}\{Y \leqslant 0\} = 0$.

Definition 1 [18]. Let Y be a fuzzy variable. The expected value of Y is defined as

$$E[Y] = \int_0^\infty \mathrm{Cr}\{Y \geqslant r\}\mathrm{d}r - \int_{-\infty}^0 \mathrm{Cr}\{Y \leqslant r\}\mathrm{d}r \tag{2}$$

provided that one of the two integrals is finite.

Particularly, for nonnegative fuzzy variable Y, $E[Y] = \int_0^\infty \mathrm{Cr}\{Y \geqslant r\}\mathrm{d}r$.

Download English Version:

https://daneshyari.com/en/article/395622

Download Persian Version:

https://daneshyari.com/article/395622

Daneshyari.com