Information Sciences 179 (2009) 2309–2316

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Defining the Borda count in a linguistic decision making context

J.L. García-Lapresta *, M. Martínez-Panero, L.C. Meneses

Departamento de Economía Aplicada, Facultad de Ciencias Económicas y Empresariales, Universidad de Valladolid, Avda. Valle de Esgueva 6, 47011 Valladolid, Spain

ARTICLE INFO

Article history:
Received 15 December 2008
Accepted 20 December 2008

Keywords:
Linguistic labels
Linguistic preferences
Trapezoidal fuzzy numbers
Borda count
Condorcet winner
Condorcet loser

ABSTRACT

Different kinds of decision rules have been successfully implemented under a linguistic approach. This paper aims the same goal for the Borda count, a well-known procedure with some interesting features. In order to this, two ways of extension from the Borda rule to a linguistic framework are proposed taking into account all the agents' opinions or only the favorable ones for each alternative when compared with each other. In the two cases, both individual and collective Borda counts are analyzed, asking for properties as good as those of the original patterns.

1. Introduction

The Borda rule is an appropriate procedure in multi-person decision making when several alternatives are considered. This estimation relies on the processed information from the whole set of alternatives, not only from each agent's most preferred one (see Saari [29, p. 19] and Dummett [10], among others). In addition, Black [3,4], Mueller [27] and Straffin [31] have noted that the Borda count chooses the alternative which stands highest on average in the agents' preference orderings.

The literature about the Borda rule is very extensive (see Martínez-Panero [25] and Richter and Wong [28] for references). Of course, the original memory of Borda [5] must be mentioned, but a more comprehensive analysis, with adjustments in the case of indifference between alternatives, can be found in Black [3,4] and Gärdenfors [17], among others. However, this treatment is not faithful enough to the agents' opinions on the alternatives, due to the way of codifying their ordinary preferences, only through discrete values, namely 0, 0.5 and 1.

There exists a natural development which allows the agents to graduate their opinions on the alternatives by means of fuzzy preferences. This approach extends the above mentioned range to all possible values from 0 to 1, and provides a more versatile count than those based on ordinary preferences. On this gradual Borda rule and its variants, see Marchant [23,24], García-Lapresta and Martínez-Panero [15,16] and Martínez-Panero [25].

The inputs of the mentioned Borda counts are numerical assessments. However, the agents tend to operate (at least in a latent manner) in terms of linguistic expressions rather than with numbers (see Zadeh [35–37] and Herrera et al. [18], among others). Well, this fact conducts us naturally to a linguistic framework. So, our aim in this paper consists in designing linguistic Borda counts from the original Borda rule and its discrete or gradual extensions, preserving their good features as far as possible (on this approach see García-Lapresta et al. [14]). Whichever the pattern may be (discrete or gradual), we note that the Borda rule is a two stage scheme in the following sense. In the first phase, individual Borda counts are computed, and it

* Corresponding author.
E-mail addresses: lapresta@eco.uva.es (J.L. García-Lapresta), panero@eco.uva.es (M. Martínez-Panero), lmeneses@eco.uva.es (L.C. Meneses).

0020-0255/$ - see front matter © 2009 Elsevier Inc. All rights reserved.
doi:10.1016/j.ins.2008.12.021

would be desirable for these scores to respect the agent's opinions on the alternatives. In García-Lapresta and Martínez-Panero [16] and Martínez-Panero [25] we have asked for the individual Borda counts to be representative of the corresponding preferences, and in connection with this aspect, different modalities of transitivity which ensure representativity for the individual counts have been analyzed, depending on the discrete or gradual version of the implemented procedure. The second phase aggregates individual scores into a total one for each alternative, and the highest score determines the Borda winner. We note that this scheme does not respect the Condorcet principle: the alternative which defeats each other by simple majority in pairwise tournaments might not be the Borda winner (in fact Gärdenfors [17] and Young [34] extend this result to any scoring rule, as Condorcet did). So, the Borda rule is not Condorcet consistent (see Baharad and Nitzan [1] for a deeper insight on this analysis). However, the Condorcet winner cannot be a Borda loser (see Fishburn and Gehrlein [12] for the original Borda rule, and Martínez-Panero [25] for the discrete and gradual cases).

The paper is organized as follows. In Section 2, we use linguistic labels for the agents to express their preferences. Then, we have represented the labels through trapezoidal fuzzy numbers, which capture the vagueness of such modality of preferences. The usual fuzzy arithmetic and a specific order are also established for computation. In Section 3, two kinds of both individual and collective linguistic Borda counts are introduced. Moreover, we find linguistic transitivity conditions which ensure these procedures be representative of the individual linguistic preferences. Section 3.2 is devoted to show, by means of an example, how to implement such linguistic Borda procedures. Section 3.3 includes the aforementioned Borda–Condorcet analysis. This is extended to a linguistic context, and a symmetry condition on the semantics associated with the set of labels is found to guarantee desirable properties for one of the introduced linguistic Borda counts. Finally, Section 4 contains some concluding remarks.

2. Linguistic preferences

Preferences are a basic instrument for dealing with decision problems. Although fuzzy preferences are a relevant tool for modelling preference intensities, linguistic preferences could be more appropriate for capturing the lack of precision in human behavior. Some papers related to the linguistic approach in decision making are Yager [33], Herrera et al. [18,19], Herrera – Martínez [20], Herrera-Viedma [21], Herrera-Viedma and Peis [22], Batyrshin et al. [2], Xu [32] and García-Lapresta [13], among others.

Now we take into account the approach included in García-Lapresta [13]. Let $X = \{x_1, \ldots, x_n\}$ be a set of alternatives and assume that m agents show their preferences over the pairs of X in a linguistic manner, with $n \geqslant 2$ and $m \geqslant 2$. Let $L = \{l_0, l_1, \ldots, l_s\}$ be a set of linguistic labels, where $s \geqslant 2$, ranked by a linear order: $l_0 < l_1 < \cdots < l_s$. There ought to be an intermediate label representing indifference, and the rest of labels are defined around it symmetrically. The number of labels, $s+1$, will be odd and, consequently, $l_{s/2}$ is the central label.

Suppose that each agent $k \in \{1, \ldots, m\}$ compares all the pairs of alternatives of X and declares levels of preference by means of a linguistic binary relation $R^k : X \times X \to L$. In what follows we use the notation $r_{ij}^k = R^k(x_i, x_j)$ and it means the level of preference with which agent k prefers x_i over x_j.

Definition 1. A *linguistic preference relation on X based on L* is a mapping $R^k : X \times X \to L$ such that:

$r_{ij}^k = l_s$, if x_i is totally preferred to x_j,
$l_{s/2} < r_{ij}^k < l_s$, if x_i is somewhat preferred to x_j,
$r_{ij}^k = l_{s/2}$, if x_i is indifferent to x_j,
$l_0 < r_{ij}^k < l_{s/2}$, if x_j is somewhat preferred to x_i,
$r_{ij}^k = l_0$, if x_j is totally preferred to x_i.

Remark 2. On the sequel we will assume that linguistic preference relations satisfy the following *reciprocity* condition:

$$r_{ij}^k = l_h \Longleftrightarrow r_{ji}^k = l_{s-h},$$

for all $x_i, x_j \in X$ and all $h \in \{0, 1, \ldots, s\}$.

The Borda count requires these labels to be added, and the results to be compared. We note that linguistic labels can be managed symbolically by means of the linguistic OWA operators introduced in Herrera et al. [19]. However, as pointed out before, in this paper we follow García-Lapresta [13]. In this way, we consider the commutative monoid $(\langle L \rangle, +)$ generated by L by means of all possible sums of labels of L with an associative and commutative operation + on L, where l_0 is the neutral element:

(1) $L \subset \langle L \rangle$
(2) $l + l' \in \langle L \rangle$, for all $l, l' \in \langle L \rangle$
(3) $l + (l' + l'') = (l + l') + l''$, for all $l, l', l'' \in \langle L \rangle$
(4) $l + l' = l' + l$, for all $l, l' \in \langle L \rangle$
(5) $l + l_0 = l$, for all $l \in \langle L \rangle$.

We also consider a total order $\leqslant$ on $\langle L \rangle$ compatible with the original order on L: