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Renewal process with T-related fuzzy inter-arrival times and fuzzy rewards

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Abstract

In this paper, we consider a renewal process in which the inter-arrival times and rewards are characterized as fuzzy variables under t-norm-based fuzzy operations. A T-related fuzzy renewal theorem and a fuzzy renewal reward theorem are proved using a law of large numbers for fuzzy variables.

Keywords: Renewal process; Fuzzy variable

1. Introduction

The theory of fuzzy sets introduced by Zadeh [17,18] has been extensively studied and applied to statistics and possibility areas in recent years. Since Kwakernaak [9,10] and Puri and Ralescu [13] introduced the concept of fuzzy random variable, there has been increasing interest in fuzzy variables. But there are only a few papers [8,12,19] investigating the renewal theory in fuzzy

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environment. Hwang [8] considered the stochastic process for fuzzy random variables and proved a theorem for fuzzy rate of a fuzzy renewal process. Popova and Wu [12] derived a theorem which presented the long-run average fuzzy reward by using the strong law of large numbers. Zhao and Liu [19] discussed the renewal process under the consideration of fuzzy inter-arrival times and proved that the expected reward per unit time was the expected value of the ratio of the reward spent in one cycle to the length of the cycle. They all of them used *min*-norm-based fuzzy operations. In general we can consider the extension principle realized by means of some *t*-norm. Renewal theory is closely related with law of large numbers. Recently, many different types of law of large numbers for *T*-related fuzzy variables have been studied by many authors, for example, [1,3,15,11,4,7,6,5,16]. In this paper, we consider a renewal reward process in which the inter-arrival times and rewards are modelled as fuzzy variables using *t*-norm-based fuzzy operations.

In Section 2, we provide some definitions and some basic results of *T*-sum of *L*–*R* fuzzy variables. In Section 3, we discuss the renewal reward processes under the consideration of fuzzy inter-arrival times and fuzzy rewards. We derive a fuzzy renewal theorem for the rate of a renewal process having fuzzy inter-arrival times and a fuzzy renewal reward theorem which present the long-run average fuzzy reward by using law of large numbers for fuzzy variables.

2. Preliminaries

As defined in [2], by a fuzzy variable we mean a function ξ of the real line with a unimodal, upper semi-continuous membership function such that there exists a unique real number m satisfying $\xi(m) = \sup_x \xi(x) = 1$. The number $m = m(\xi)$ is called the modal value of ξ. Now suppose that a sequence of fuzzy variables $\xi_1, \xi_2, \ldots, \xi_n, \ldots$ and a *t*-norm T (see [20]) are given. The *T*-sum $\xi_1 + \xi_2 + \cdots + \xi_n$ and the *T*-arithmetic mean $(\xi_1 + \xi_2 + \cdots + \xi_n)/n$ are the fuzzy variables defined by

$$\xi_1 + \xi_2 + \cdots + \xi_n(z) := \sup_{x_1 + x_2 + \cdots + x_n = z} T(\xi_1(x_1), \ldots, \xi_n(x_n))$$

and

$$\frac{1}{n}(\xi_1 + \xi_2 + \cdots + \xi_n)(z) := (\xi_1 + \xi_2 + \cdots + \xi_n)(nz),$$

respectively (see [2]). It is easy to see that

$$\begin{aligned} m(\xi_1 + \xi_2 + \cdots + \xi_n) &= m(\xi_1) + m(\xi_2) + \cdots + m(\xi_n) \\ &= nm\left(\frac{1}{n}(\xi_1 + \xi_2 + \cdots + \xi_n)\right). \end{aligned}$$