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Fuzzy rough set theory for the interval-valued fuzzy information systems ☆

Bingzhen Sun [a], Zengtai Gong [b,*], Degang Chen [c]

[a] School of Traffic and Transportation, Lanzhou Jiaotong University, 730070, PR China
[b] College of Mathematics and Information Science, Northwest Normal University, Lanzhou 730070, PR China
[c] Department of Mathematics and Physics, North China Electric Power University (Beijing), Beijing 102206, PR China

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Abstract

The concept of the rough set was originally proposed by Pawlak as a formal tool for modelling and processing incomplete information in information systems, then in 1990, Dubois and Prade first introduced the rough fuzzy sets and fuzzy rough sets as a fuzzy extension of the rough sets. The aim of this paper is to present a new extension of the rough set theory by means of integrating the classical Pawlak rough set theory with the interval-valued fuzzy set theory, i.e., the interval-valued fuzzy rough set model is presented based on the interval-valued fuzzy information systems which is defined in this paper by a binary interval-valued fuzzy relations $R \in F^{(i)}(U \times U)$ on the universe U. Several properties of the rough set model are given, and the relationships of this model and the others rough set models are also examined. Furthermore, we also discuss the knowledge reduction of the classical Pawlak information systems and the interval-valued fuzzy information systems respectively. Finally, the knowledge reduction theorems of the interval-valued fuzzy information systems are built.

Keywords: Fuzzy sets; Rough sets; Interval-valued fuzzy sets; Interval-valued fuzzy information systems; Knowledge reduction

1. Introduction

The theory of rough sets was firstly proposed by Pawlak [17–19]. It is an extension of the set theory for the study of intelligent systems characterized by insufficient and incomplete information. The successful application of rough set theory in a variety of problems has been amply demonstrated its usefulness. This theory evoked into a far-reaching methodology centering on analysis of incomplete information [9,23,24,31]. It soon evoked a natural question concerning possible connections between rough sets and fuzzy sets. Generally

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* Corresponding author. Tel.: +86 931 7971845.
E-mail addresses: zt-gong@163.com, gongzt@nwnu.edu.cn (Z. Gong).

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speaking, both theories address the problem of information granulation: the theory of fuzzy sets is centrad upon fuzzy information granulation, whereas rough set theory is focused on crisp information granulation. Originally, the basic notion in rough set theory was indiscernibility (i.e., indiscernibility between objects in information systems induced by different values of attributes characterizing these objects), yet in recent extensions [7,14,20,22] the focus moves to the notion of similarity, which is in fact a fuzzy concept. Therefore, it is apparent that these two theories have become much more closely related to each other.

A key notion in Pawlak's rough set model is equivalence relation. The equivalence classes are the building blocks for the construction of the lower and upper approximations. By replacing the equivalence relation with an arbitrary binary relation, different kinds of generalizations in Pawlak's rough set models were obtained. Dubois and Prade [3,4] were among the first who investigated the problem of a fuzzyfication of a rough set, the concept of rough fuzzy set and fuzzy rough set were proposed by replacing crisp binary relations with fuzzy relations in the universe [1,3,4,12,13,25,32]. Moreover, Dubois and Prade also pointed out that the rough fuzzy set is a special case of the fuzzy rough set in the universe in their literatures. The fuzzy rough set theory, proposed by the others authors, has been made up the deficiencies of the traditional rough set theory in several aspects. However, there are both of the symbolic values, real values and possibly interval values of the attributes in the real life database [6], therefore, the traditional fuzzy rough set theory could not deal with those data effectively. It is then necessary to extend the traditional fuzzy rough set theory in a general sense. In this paper, we propose the interval-valued fuzzy rough set theory by combining the interval-value fuzzy set theory with the traditional rough set theory. Therefore, the traditional fuzzy rough set theory is extended and its weaknesses are overcome.

There are at least two approaches for the development of the fuzzy rough sets theory, the constructive and the axiomatic approaches [29,30,33]. Moris and Yakout [11] provide the axiomatization for the fuzzy rough set model. Wu et al. [28] present a general framework for the study of fuzzy rough sets in which both constructive and axiomatic approaches are used. In the constructive approach, the relations in the universe is the primitive notion, the lower and upper approximation operators are constructed by means of this notion, therefore, one can obtain a pair of lower and upper generalized approximation operators in this approach. Dubois and Prade were among the first researchers to propose the concept of the fuzzy rough sets from the constructive approach. In the axiomatic approach, various classes of fuzzy rough approximation operators are characterized by different sets of axioms, these axioms guarantee the existence of certain types of fuzzy relations producing the same operators. This paper is devoted to the discussion of the interval-valued fuzzy rough sets model by the constructive approach.

Fuzzy rough sets have been used to solve practical problems such as data mining [9,16], approximate reasoning [21], medical time series, case generation [15], mining stock price [27], and descriptive dimensionality reduction [6].

As a generalization of the Zadeh fuzzy set, the notion of interval- valued fuzzy sets was suggested for the first time by Gorzalczany [5] and Turksen [26], and it was applied to the fields of approximate inference, signal transmission and controller, etc. In this paper we combine the classical Pawlak rough sets theory with the interval-valued fuzzy sets theory, define the interval-valued fuzzy information system, and discuss the rough set theory of the interval-valued fuzzy information system. So, the interval-valued fuzzy rough sets model is obtained in the interval-valued fuzzy information system by the constructive approach. Several properties of this model are given, and the relationships of this model and the other rough set models are also examined.

Knowledge reduction [2] is performed in information systems by means of the notion of a reduction based on a specialization of the general notion of independence due to Maczewski [10]. The knowledge reduction of consistent information systems based on the rough sets theory have been many practices conclusion. In recent years, more attention has been paid to knowledge reduction in inconsistent systems in rough sets research [8]. Many types of knowledge reduction have been proposed in the area of rough sets [30,31]. In this paper, we are concerned with approaches to knowledge reduction based on the interval-valued fuzzy rough sets model. We first define the interval-valued fuzzy reduction on the classical Pawlak information systems, then discuss the knowledge reduction of the interval-valued fuzzy information systems. Finally, the knowledge reduction theorems of the interval-valued fuzzy information systems are built.

The structure of the rest of this paper is as follows: Section 2 briefly introduces necessary notions of rough sets, fuzzy sets and interval-valued fuzzy sets. In Section 3, we define the interval-valued fuzzy binary relations