Information Sciences 180 (2010) 1121–1133

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Image registration by compression

Anton Bardera *, Miquel Feixas, Imma Boada, Mateu Sbert

Graphics and Imaging Laboratory, University of Girona, 17071 Girona, Spain

ARTICLE INFO

Article history:
Received 15 February 2008
Received in revised form 10 September 2009
Accepted 21 November 2009

Keywords:
Image registration
The similarity metric
Kolmogorov complexity
Information theory

ABSTRACT

Image registration consists in finding the transformation that brings one image into the best possible spatial correspondence with another image. In this paper, we present a new framework for image registration based on compression. The basic idea underlying our approach is the conjecture that two images are correctly registered when we can maximally compress one image given the information in the other. The contribution of this paper is twofold. First, we show that image registration can be formulated as a compression problem. Second, we demonstrate the good performance of the similarity metric, introduced by Li et al., in image registration. Two different approaches for the computation of this similarity metric are described: the Kolmogorov version, computed using standard real-world compressors, and the Shannon version, calculated from an estimation of the entropy rate of the images.

1. Introduction

Image registration is an important focus of research in image processing. Registration consists in matching two or more images in a common coordinate system. A usual method of solving the registration task is to treat it as a mathematical optimization problem, using a similarity measure to quantify the quality of the alignment of the two images for any given transformation. Hence, the task of finding out the correct registration between two images is frequently based on the maximization of a similarity measure or the minimization of a given distance. Some information-theoretic measures, such as *mutual information* (*MI*) [24,36] and *normalized mutual information* (*NMI*) [34], have become a standard reference, mainly in medical imaging, due to their accuracy and robustness [27,25].

In this paper, an extended and revised version of earlier work [1], the *normalized information distance* (*NID*), also called *the similarity metric*, is proposed as a new similarity measure for image registration. Introduced by Li et al. [22] for measuring similarity between sequences, *NID* is based on the non-computable notion of *Kolmogorov complexity* and is a normalized version of the information distance [3]. In essence, the main idea behind it is that two objects are similar if we can significantly compress one, given the information in the other. This measure has been successfully applied in different areas such as genome phylogeny [21], language phylogeny [22], and classification of music pieces [7].

However, the application of *NID* is limited by its non-computability. To tackle this problem, we propose two different approaches. The first one is based on the *normalized compression distance* (*NCD*) [8], which approximates the Kolmogorov complexity using real-world compressors. In this case, the capability of the compressor to approximate the Kolmogorov complexity will determine the registration accuracy. The second approach is based on the *normalized entropy rate distance* (*NED*) [19,13], which substitutes the Kolmogorov complexity by the entropy rate. This is a measure of the degree of compressibility of an image from a Shannon perspective. In both cases, experimental results demonstrate that *the similarity*

* Corresponding author.
E-mail address: anton.bardera@ima.udg.edu (A. Bardera).

0020-0255/$ - see front matter © 2009 Elsevier Inc. All rights reserved.
doi:10.1016/j.ins.2009.11.031

metric performs well in image registration, although the Kolmogorov version is less accurate and robust than the entropy rate approach due to the compressor imperfections.

This paper is organized as follows. In Section 2, we survey background and related work. In Section 3 we present our compression-based framework for image registration. Sections 4 and 5 introduce the proposed registration approaches based on Kolmogorov complexity and entropy rate, respectively. Experimental results are given in Section 6 and, finally, the conclusions and future work are summarized in Section 7.

2. Background

In this section we review *the similarity metric* based on the Kolmogorov complexity [22], some basic information-theoretic measures [10,14], and their application to image registration [24,36,27].

2.1. The similarity metric

The *Kolmogorov complexity* $K(x)$ of a string x is the length of the shortest program to compute x on an appropriate universal computer.[1] Essentially, the Kolmogorov complexity of a string is the length of the ultimate compressed version of the string. The conditional complexity $K(x|y)$ of x relative to y is defined as the length of the shortest program to compute x given y as an auxiliary input to the computation. The joint complexity $K(x,y)$ represents the length of the shortest program for the pair (x,y) [22]. For a detailed review see [23].

In [3], the *information distance* is defined as the length of the shortest program that computes x from y and y from x. It was shown there that, up to an additive logarithmic term, the information distance is given by

$$E(x,y) = \max\{K(y|x), K(x|y)\}. \tag{1}$$

It was also shown that $E(x,y)$ is a metric. It is interesting to note that long strings that differ by a tiny part are intuitively closer than short strings that differ by the same amount. Hence, there arises the necessity to normalize the information distance. In [22], the normalized version of $E(x,y)$, called the *normalized information distance* (*NID*) or *the similarity metric*, is defined by

$$NID(x,y) = \frac{\max\{K(x|y), K(y|x)\}}{\max\{K(x), K(y)\}} = \frac{K(x,y) - \min\{K(x), K(y)\}}{\max\{K(x), K(y)\}}. \tag{2}$$

Li et al. show that $NID(x,y)$ is a metric and it takes values in $[0,1]$. This metric is universal in the sense that if two strings are similar according to the particular feature described by a particular normalized admissible distance (not necessarily metric), then they are also similar in the sense of the normalized information metric [8].

The Kolmogorov complexity K is a non-computable measure in the Turing sense [23] and, therefore, for real-world applications, we will need an approximation of it. An upper bound of the non-computable complexity K is the length of compressed string x (or y), $C(x)$ (or $C(y)$), generated by a compression algorithm. The better the compression algorithm, the better the approximation to K. Then, a feasible version of the normalized information distance (2), called the *normalized compression distance* (*NCD*), is defined [8] as

$$NCD(x,y) = \frac{C(x,y) - \min\{C(x), C(y)\}}{\max\{C(x), C(y)\}}, \tag{3}$$

where $C(x)$ (or $C(y)$) represents the length of compressed string x (or y) and $C(x,y)$ the length of the compressed pair (x,y). Thus, *NCD* is computed from the lengths of compressed data files and, therefore, *NCD* approximates *NID* by using standard real-world compressors.

2.2. Information-theoretic measures

Let $\mathcal{X}$ be a finite set, let X be a random variable taking values x in $\mathcal{X}$ with distribution $p(x) = Pr[X = x]$. Likewise, let Y be a random variable taking values y in $\mathcal{Y}$. The *Shannon entropy* $H(X)$ of a random variable X is defined by

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log p(x). \tag{4}$$

The Shannon entropy $H(X)$ measures the average uncertainty of random variable X. If the logarithms are taken in base 2, entropy is expressed in bits. The *conditional entropy* is defined by

[1] A universal computer, or Turing machine, is a theoretical computing machine, invented by Alan Turing [35], to serve as an idealized model for mathematical calculation. A Turing machine consists of a line of cells known as a "tape" that can be moved back and forth, an active element known as the "head" that possesses a property known as "state" and that can change the property known as "color" of the active cell underneath it, and a set of instructions for how the head should modify the active cell and move the tape [37]. In our context, the shortest program to compute x on an appropriate universal computer is equivalent to consider the length of the shortest binary program to compute x in a universal programming language, such as Java [22].