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Parameterized defuzzification with continuous weighted quasi-arithmetic means – An extension ☆

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ABSTRACT

Parameterized aggregation becomes an attractive idea in many applications. By applying the quasi-arithmetic mean-based defuzzification method and its orness measure, we extend the properties of discrete quasi-arithmetic mean with an orness measure to the case of continuous weighted quasi-arithmetic means, and apply them to the parameterized estimation of fuzzy or random variables. When facing the uncertainty of these variables, the decision maker can obtain the crisp value estimation which is always consistent with his/her preference information. Two families of continuous weighted quasi-arithmetic means with exponential and power function generators are discussed. Combining these conclusions with that of the discrete case, we can understand better the properties of quasi-arithmetic means and extend their possible applications.

1. Introduction

Parameterized aggregation methods which can be tuned to become *or-like* (close to the maximum) or *and-like* (close to the minimum) have been an active topic in recent years [1,3,7,8,12,18,32,41,45], and have become important tools in the uncertainty theory and computing with words method [19,29,50,52,55,56]. They include parameterized t-norms and t-conorms, ordered weighted averaging operator aggregation, quasi-arithmetic means and the operators based on Choquet and Sugeno integrals. Among them, the ordered weighted averaging (OWA) operator which was introduced by Yager [46], has attracted much interest among researchers [2,17,40,45,53].

Another class of parameterized aggregation operators commonly used in theory and applications is the quasi-arithmetic mean [16,20,21,30,33,35,37,42,43], which covers a wide spectrum from arithmetic, quadratic, geometric and harmonic to the general root-power and exponential means. It has been proved that the quasi-arithmetic means are the only symmetric, continuous, strictly increasing, idempotent, real functions which satisfy the bisymmetry condition [30, pp. 33–34].

One of the appealing points in the OWA operators is the concept of *orness* [46], which is very important both in theory and applications [15,44,48,49]. The extension of Yager's *orness* concept to other aggregation operators has recently been focused on by some researchers. The measure of orness was first introduced by Dujmović [13] for the power means under the name of disjunction degree. Marichal [30] proposed that the degrees of orness can be defined for any compensative aggregation operator. Calvo et al. [5] suggested an extension of OWA operators by applying the concept of weighting triangles in place of the standard OWA weights. Recently, Salido and Murakami [34] extended the OWA orness measure to fuzzy aggregation operators. Larsen [21] also proposed an orness measure for a special type of root-power mean in the quasi-arithmetic mean

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family. Liu [24] proposed an orness measure for quasi-arithmetic means and discussed its properties. Two kinds of quasi-arithmetic means with exponential and power function generator were discussed. An interesting property of these two quasi-arithmetic mean families is that the aggregation values for any set always monotonically increase with the orness level, that they can be used as parameterized aggregation processes.

In many aggregation applications, in addition to the satisfaction scores of the criteria, we also have importance indexes associated with the criteria. The importance can be the subjective information such as the weights assigned by the decision maker or the objective information such as the probability distribution. Some aggregation methods with importance or weights information have been proposed for OWA operators, quasi-arithmetic means, and other aggregation operators [6,9–11,14,16,21,33,35–38,47]. These weighted aggregation methods have been used for the fuzzy number estimation or defuzzification in recent years [14,22,23,25–28,31,39,47,51]. If the membership function of a fuzzy set is replaced with the probability density function, these methods can also be used to estimate the random variables.

In the present paper, inspired by the the work of Yoshida [54] for the fuzzy number defuzzification with continuous weighted quasi-arithmetic means, we will extend the orness measure properties of discrete quasi-arithmetic means [24] to the case of continuous weighted quasi-arithmetic mean, and will apply them to the parameterized preference defuzzification of fuzzy sets or the parameterized preference expectation of random variables, where orness is used as the control variable or preference information representation parameter. Despite the extensions to the results of [24], these conclusions can also be seen as an extension of [54], which make us better understand the properties of quasi-arithmetic means for further possible applications.

2. Defuzzification with continuous weighted quasi-arithmetic mean operator

The fuzzy number defuzzification method with continuous weighted quasi-arithmetic means was proposed by Yoshida recently [54] in the domain $(-\infty, +\infty)$. Some properties of it were discussed. Here, we will use this method with the ordinary fuzzy set in a compact interval. All the conclusions can be extended to the ordinary fuzzy set in $(-\infty, +\infty)$ directly. If $\mu_A(x)$ is replaced with a probability density function of a random variable $p_\xi(x)$, the method in this paper can also be seen as an estimation of the random variable ξ.

A fuzzy set A is characterized by a generalized characteristic function $\mu_A : X \to [0,1]$, called membership function of A and it is defined over a universe of discourse X. We restrict X to be a bounded subset of the real line. The set of all elements that have a nonzero degree of membership in A is called the *support* of A, i.e.

$$\mathrm{supp}(A) = \{x \in X | \mu_A(x) > 0\}. \tag{1}$$

The set of elements with the largest degree of membership in A is called the *core* of A, i.e.

$$\mathrm{core}(A) = \left\{x \in X | \mu_A(x) = \sup_{x\in X} \mu_A(x)\right\}. \tag{2}$$

In the following, we will always assume that A is continuous and with a bounded support $\mathrm{supp}(A) = (a,b)$. The strong support of A should be $\mathrm{supp}'(A) = [a,b]$.

As a special case, the fuzzy number A with membership function $\mu_A(x)$ is a fuzzy set of the real line $\mathbb{R}$ with a normal, convex and upper-semicontinuous membership function of a bounded support set. Some special forms of fuzzy numbers are preferred in practice with simple membership functions and natural interpretations. The most often used fuzzy numbers are trapezoidal fuzzy numbers with membership function in (3):

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < a_1, \\ \frac{x-a_1}{a_2-a1} & \text{if } a_1 \leqslant x < a_2, \\ 1 & \text{if } a_2 \leqslant x \leqslant a_3, \\ \frac{a_4-x}{a_4-a_3} & \text{if } a_3 < x \leqslant a_4, \\ 0 & \text{if } x > a_4. \end{cases} \tag{3}$$

If $a_2 = a_3$, A becomes a triangular number; furthermore, if $a_1 = a_2$, $a_3 = a_4$, A becomes an interval number; and if $a_1 = a_2 = a_3 = a_4$, A becomes a crisp real number.

With Zadeh's extension principle, the arithmetic operations of fuzzy sets especially the fuzzy numbers can be defined. Here, we only recall the two simplest cases of scalar addition and scalar multiplication. For the fuzzy set with membership function $\mu_A(x)$, the membership function of scalar addition $A + c$ and scalar multiplication $kA (k \neq 0)$ are $\mu_{A+c}(x) = \mu_A(x - c)$ and $\mu_{kA} = \mu_A(x/k)$, respectively.

Definition 1. Let f be a continuous strictly monotonic mapping on $[a,b]$. For aggregated elements vector $X = (x_1, x_2, \ldots, x_n) \in [a,b]^n$, a quasi-arithmetic mean can be defined as the aggregation operator $M_f : [a,b]^n \to [a,b]$ with

$$M_f(x_1, x_2, \ldots, x_n) = f^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} f(x_i)\right), \tag{4}$$

where f^{-1} is its inverse function. f is called the generator of the quasi-arithmetic mean M_f. We will denote this with $M_f(X)$.