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H_∞ filtering of networked discrete-time systems with random packet losses

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ABSTRACT

This paper studies the H_∞ filtering problem for networked discrete-time systems with random packet losses. The general multiple-input–multiple-output (MIMO) filtering system is considered. The multiple measurements are transmitted to the remote filter via distinct communication channels, and each measurement loss process is described by a two-state Markov chain. Both the mode-independent and the mode-dependent filters are considered, and the resulting filtering error system is modelled as a discrete-time Markovian system with multiple modes. A necessary and sufficient condition is derived for the filtering error system to be mean-square exponentially stable and achieve a prescribed H_∞ noise attenuation performance. The obtained condition implicitly establishes a relation between the packet loss probability and two parameters, namely, the exponential decay rate of the filtering error system and the H_∞ noise attenuation level. A convex optimization problem is formulated to design the desired filters with minimized H_∞ noise attenuation level bound. Finally, an illustrative example is given to show the effectiveness of the proposed results.

1. Introduction

Many modern engineering systems with expanding physical setups and functionality are now experiencing a period of changing their architectures from the traditional centralized point-to-point ones to the decentralized ones. The progress in computer and communication technologies makes it to be possible to transmit signals among sensors, controllers, estimators, and actuators over communication networks in these decentralized modern engineering systems. Filtering systems transmitting signals over communication networks are called network-based filtering systems. Such network-based systems bring us lots of advantages, and have received increasing research attention, see for example, [4,6–9,12,13,19,20,23,25,26] and the references therein. However, communication networks are usually unreliable, and may be subject to undesirable packet losses and network-induced delays, which may significantly degrade the system performance. Therefore, the negative effects caused by communication networks should be taken into account in designing such network-based systems to obtain desired filtering performance.

During the last decade, a few results have been reported on the network-based filtering problem [5,11,14,17,18,21], and most of them deal with the packet loss issue. Markov chain and independent Bernoulli random process were commonly used to model the packet loss processes. The results applying Bernoulli process assume that the packet losses are independent and are identically distributed [5,21], while those using Markov chain assume that the packet losses are bursty and occur according to a Markov chain [11,17,18]. In most of the existing results concerning filtering of network-based systems, the filter inputs are set to zero when the measurements are lost. With this setup, the signal estimation with packet losses is usually called estimation with missing or intermittent observations [5,17,18,21].

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Most of the aforementioned results adopted the Kalman filtering method. Though the celebrated Kalman filtering is commonly regarded as one of the most popular and useful approaches to filtering problem, it usually assumes that the system model is precise, and that the statistical information of external noises is exactly known. Such assumptions may not hold in many practical applications. In this case, one may resort to other useful filtering algorithms. H_∞ filtering is among these useful algorithms, and it provides both a guaranteed noise attenuation level and robustness against model uncertainties [1]. The H_∞ filtering for network-based systems has appeared recently in [1,15,22,24]. In [22], the H_∞ filter was designed for a class of networked continuous-time linear systems with delays, packet losses and model uncertainties. By considering discrete-time linear plants, the optimal H_∞ filter was designed in [15] for a class of networked systems with consecutive packet losses. In [1], the H_∞ filtering problem was studied for a class of networked systems with packet losses, delays and quantization effects.

On the other hand, in a network-based filtering system, the various sensors and filters are usually distributed in a wide area, and the multiple measurements of the plant cannot be encapsulated into one single packet and must be transmitted to the remote filter via distinct channels. In this case, the traditional assumption that the multiple measurements are transmitted through one common channel or that the packet loss processes in different channels are identical may not hold. Therefore, it is necessary to develop a filtering algorithm for network-based systems with multi-channel transmission mechanism. Furthermore, one may expect to establish a relation between the packet loss probability and the filtering performance (such as the stability performance of the filtering error system and the noise attenuation performance). Such a relation may provide us some useful guidelines to design a filter with desired filtering performance. However, few results in the existing literature are concerned with this topic. This is the motivation of the present research.

In this paper, the H_∞ filtering problem is investigated for a class of networked discrete-time linear systems with random packet losses. Different from the existing formulation for the estimation problem with missing observations, we assume that the filter input is held at its previous values when current measurements are lost. A two-state Markov chain is used to describe the random packet loss process in each communication channel. Then, the main contributions of the obtained results are as follows: (1) a general model is proposed to describe the filtering error system with multiple packet losses. The obtained system model is more practical than some existing ones in the case that the sensors are separately distributed and the measurements cannot be encapsulated into one data packet. Moreover, the presented system model can also be used to describe a networked filtering system with all measurements transmitted through one common communication channel; (2) a necessary and sufficient condition is derived for the filtering error system to be mean-square exponentially stable and to achieve a prescribed H_∞ performance level. Moreover, the obtained condition is decay-rate-dependent, where the decay rate is a parameter that can be tuned according to the packet loss status in networks. Thus, the obtained condition implicitly establishes a relation between packet loss probability and performance level of the filtering system. As a special case, the obtained condition is reduced to the BRL (Bounded Real Lemma) in Theorem 2 in [16] when the decay rate therein is limited to 1 and all the measurements are assumed to be transmitted through one common channel; (3) by considering both mode-independent and mode-dependent filters, the filtering error system is modelled as a discrete-time Markovian switched system with multiple modes. The mode-dependent filters are designed to cope with different packet loss situations in networks, and thus to reduce the conservatism of the filter design. A convex optimization problem is also formulated to design the filters with minimized H_∞ noise attenuation level. Finally, an illustrative example is given to demonstrate the effectiveness of the proposed results.

2. Problem statements and preliminaries

Consider a plant described by the following discrete-time linear system model:

$$\begin{aligned} S: x(k+1) &= A_p x(k) + B_p w(k) \\ y(k) &= C_p x(k) + D_p w(k) \\ z(k) &= L_p x(k) \end{aligned} \tag{1}$$

where $x(k) \in \Re^n$ is the system state, $y(k) \in \Re^m$ is the measured output, $z(k) \in \Re^p$ is the signal to be estimated, and $w(k) \in \Re^q$ is the noise signal which belongs to $l_2[0,\infty)$. A_p, B_p, C_p, D_p, and L_p are constant matrices of appropriate dimensions, and A_p is assumed to be stable. The estimation problem under consideration is illustrated in Fig. 1, where $y_i(k), i = 1,\ldots,m$ are the measured outputs of the plant, and $\hat{y}_i(k)$ are the inputs of the full order filter with $\hat{y}(k) = [\hat{y}_1(k)\ldots\hat{y}_m(k)]^T$. The filter is connected to the plant via communication networks which may be subject to random packet losses, and several Markov chains are adopted to describe the packet loss processes. Assume that the sensor measurements $y_i(k), i = 1,\ldots,m$ are transmitted to the filter through m distinct communication channels. We use m two-state Markov chains $\sigma_i(k), i = 1,\ldots,m$ to describe, respectively, the random packet loss processes in the m channels, and $\sigma_i(k)$ takes values in $\Phi = \{0,1\}$. It is assumed that there is no packet loss in the ith communication channel if $\sigma_i(k) = 1$, while there is a packet loss if $\sigma_i(k) = 0$. Denote the transition probability matrix of $\sigma_i(k)$ by $N_i = (\pi_{r_i s_i}) \in \Re^{2\times 2}$, where the transition probabilities $\pi_{r_i s_i}$ are defined as follows:

$$\begin{aligned} &\pi_{r_i s_i} = \Pr(\sigma_i(k+1) = s_i | \sigma_i(k) = r_i), \quad \pi_{r_i s_i} \geqslant 0 \\ &\pi_{r_i 0} + \pi_{r_i 1} = 1, \quad \text{for all } r_i \in \Phi, \quad i = 1,\ldots,m \end{aligned}$$