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A unified methodology for the efficient computation of discrete orthogonal image moments

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ABSTRACT

A novel methodology is proposed in this paper to accelerate the computation of discrete orthogonal image moments. The computation scheme is mainly based on a new image representation method, the image slice representation (ISR) method, according to which an image can be expressed as the outcome of an appropriate combination of several non-overlapped intensity slices. This image representation decomposes an image into a number of binary slices of the same size whose pixels come in two intensities, black or any other gray-level value. Therefore the image block representation can be effectively applied to describe the image in a more compact way. Once the image is partitioned into intensity blocks, the computation of the image moments can be accelerated, as the moments can be computed by using decoupled computation forms. The proposed algorithm constitutes a unified methodology that can be applied to any discrete moment family in the same way and produces similar promising results, as has been concluded through a detailed experimental investigation.

1. Introduction

Although moments have been used in image interpretation and analysis for a long time now, scientists are still very interested in the matter. Image moments have been used successfully in image processing [6,8] and pattern recognition [11,20] after image normalization and proper selection [2,5]. By using the geometrical, central and normalized image moments [8], Hu [4] constructed seven measurements that are invariant to any translation, scaling and rotation transformation of the image being processed.

However, the Hu invariants and geometric moments suffer from high information redundancy. More precisely, the geometric moments of an image are the projections of the intensity function of the image onto specific monomials, which do not construct an orthogonal basis.

Orthogonal moments were developed to overcome this disadvantage of the conventional moments. Since their kernels are orthogonal polynomials, orthogonal moments have the ability to fully reconstruct the image described. The first introduction of orthogonal moments in image analysis, due to Teague [16], made use of Legendre and Zernike moments in image processing. Other families of orthogonal moments have been proposed over the years, such as Pseudo-Zernike, Fourier-Mellin, etc. moments, which better describe the image in process and ensure robustness under arbitrarily intense noise levels.

However, these orthogonal moments, which have been used until recently, present some approximation errors due to the fact that the kernel polynomials are defined in a continuous space [7,21]. Therefore, when the moments of a discrete intensity function must be computed, they produce some errors that influence the final results.

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Apart from some remarkable attempts to compute the theoretical image moment values [3,18,22], new moment families with discrete polynomial kernels have been proposed [9,23–26], which permit the direct computation of the image moments in the discrete domain.

Both continuous and discrete orthogonal moments present quite similar computational difficulties, since there are a lot of time-consuming quantities to be computed and it is necessary to evaluate the kernel polynomials for each pixel of the image at hand. While in the first case, many works have introduced efficient algorithms [11,12,17,19] that accelerate the computation procedure by simplifying or recursively computing the orthogonal polynomials, little attention has been paid to the second case.

The main drawback of the previously proposed algorithms is that none of the methodologies can be applied to all moment families, since they are making use of specific properties of their kernel polynomials. Thus, there is still a need for a unified method that can be effectively applied in a common way to as many as possible moment families, independently of the moment kernel.

The present paper attempts to establish a unified methodology for the computation of discrete orthogonal image moments (DOIMs), by adopting a novel algorithm introduced by the authors [13] in the case of geometrical moments. Based on the proposed method, a gray-scale image is initially decomposed into specific intensity slices by using the ISR representation [13]. The extracted intensity slices are images of the same size as the original image, whose pixels have only two different intensity values, black or an intensity value in the range [1]. Therefore, each bilevel slice can be considered as a binary image that can be described by a set of homogenous orthogonal blocks, by using the IBR algorithm [15]. The representation of an image by a set of blocks has the potential to simplify the computation formulas of the moments, as already presented in [13,15]. This two-stage methodology is adopted and applied to several discrete orthogonal moment families, by establishing a unified acceleration technique.

A significant advantage of the proposed methodology, apart from the high speed of computation, is that it can be easily applied to all discrete orthogonal moments, in the same way. Moreover, the method introduced here can improve the reconstruction procedure by accelerating the computation of the image's pixels.

The experiments show that the proposed methodology works as fast or faster than the conventional algorithms, in the worst case for gray-scale images, and it outperforms them fully in the case of binary images.

The paper is organized as follows: Section 2 describes the discrete orthogonal moment families recently introduced, in which the proposed methodology, which is analysed in Section 3, is applied. The reformulated computation schemes are derived in Section 4, while a detailed experimental study demonstrates the efficiency of the novel algorithm, in terms of its computation time, in Section 5. Finally, the conclusions of this work are discussed in Section 6.

2. Discrete orthogonal image moments (DOIMs)

Due to the limitations of the continuous image moments relative to their accuracy, discrete orthogonal moments have attracted the attention of the scientific community in recent years.

A general formulation of the $(n + m)$th order discrete orthogonal image moment of an $N \times N$ image with intensity function $f(x,y)$ is given as follows:

$$M_{nm} = NormFactor * \sum_{x=1}^{N} \sum_{y=1}^{N} Poly_n(x) Poly_m(y) f(x,y) \tag{1}$$

where *NormFactor* is a normalization factor and $Poly_n(i)$ is the nth order discrete orthogonal polynomial value of the pixel point with coordinate i, used as a moment kernel. According to the type of the polynomial kernel used in (1), the type of DOIMs such as Tchebichef, Krawtchouk, Racah and dual Hahn, is determined by the type of polynomials that are used.

The next sections provide a brief description of the Tchebichef, Krawtchouk, Racah and dual Hahn DOIMs. The DOIMs we investigate are defined with respect to generalized hypergeometric functions.

2.1. Tchebichef moments

Tchebichef moments, were the first type of discrete orthogonal moments introduced in image analysis by Mukundan in [9]. These moments use as their kernel, the Tchebichef orthogonal polynomials defined in the discrete domain and have the following form

$$t_n(x) = (1-N)_n {}_3F_2(-n, -x, 1+n; 1, 1-N; 1) = \sum_{k=0}^{n} (-1)^{n-k} \binom{N-1-k}{n-k} \binom{n+k}{n} \binom{x}{k} \tag{2}$$

where $presub3F_2$, is the generalized hypergeometric function, $n, x = 0, 1, 2, \ldots, N-1$, and N the image size.