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λ-Statistical limit points of the sequences of fuzzy numbers

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Abstract

Aytar has introduced the concepts of statistical limit and cluster points of a sequence of fuzzy numbers based on the definitions given in Fridy's study for sequences of real numbers. In this paper, we define λ-statistical limit and λ-statistical cluster points of sequences of fuzzy numbers and discuss the relations among the sets of ordinary limit points, λ-statistical limit points and λ-statistical cluster points of sequences of fuzzy numbers.

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1. Introduction

The idea of statistical convergence of a sequence was introduced by Fast [6]. Statistical convergence was generalized by Buck [4].

Fridy [7] was the first not only to introduce the set Γ_X of all statistical cluster points and the set Λ_X of all limit points but also to discuss their definitions and properties, as well as the specific relations between them and the relations to the set L_X of all ordinary limit points. These issues have been further explored in finite dimensional spaces by Pehlivan and Mamedov [8,11]. Bounded and convergent sequences of fuzzy numbers were first introduced by Matloka [9]. He also showed that every convergent sequence is bounded.

Later, Nuray and Savas [10] have introduced and discussed the concepts of statistically convergent and statistically Cauchy sequences of fuzzy numbers. Also Savas [12] has studied the λ-statistical convergence of the sequences of fuzzy numbers. λ-Statistically Cauchy sequences of fuzzy numbers have been introduced by Tuncer and Benli [13].

Recently, Aytar [1] has defined statistical limit and cluster points of a sequence of fuzzy numbers. Aytar and Pehlivan [2] have introduced the statistical monotonicity and boundedness of a sequence of fuzzy numbers.

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Aytar et al. [3] have also extended the concepts of statistical limit superior and limit inferior to statistically bounded sequences of fuzzy numbers. Moreover, they have given some fuzzy-analogues of properties of statistical limit superior and limit inferior for sequences of real numbers.

Et, Altinok and Colak [5] have introduced the concept of strongly $\Delta^2_{\lambda_p}$-Cesaro summability of a sequence of fuzzy numbers.

In this paper, as in the case of real numbers, the concepts of λ-thin and λ-nonthin subsequences of a sequence of fuzzy numbers have been given. With the help of λ-thin and λ-nonthin subsequences, we have defined λ-statistical limit and λ-statistical cluster points of a sequence of fuzzy numbers. Later, we have established the inclusion relations between the sets of ordinary limit points, λ-statistical limit points and λ-statistical cluster points of a sequence of fuzzy numbers.

2. Preliminaries

Let D denote the set of all closed bounded intervals $A = [\underline{A}, \overline{A}]$ on the real line R. For $A, B \in D$ define

$$A \leqslant B \Longleftrightarrow \underline{A} \leqslant \underline{B} \quad \text{and} \quad \overline{A} \leqslant \overline{B}$$
$$d(A,B) = \max(|\underline{A} - \underline{B}|, |\overline{A} - \overline{B}|)$$

It is easy to see that d defines a Hausdorff metric on D and (D, d) is a complete metric space. Also $\leqslant$ is a partial order on D.

A fuzzy number is a fuzzy subset of the real line R which is bounded, convex and normal. Let $L(R)$ denote the set of all fuzzy numbers which are upper semicontinuous and have compact support. In other words, if $X \in L(R)$, then for any $\alpha \in [0,1]$, X^α is compact, where

$$X^\alpha = \begin{cases} t : X(t) \geqslant \alpha & \text{if } \alpha \in (0,1] \\ t : X(t) > 0 & \text{if } \alpha = 0 \end{cases}$$

$X^\alpha = [\underline{X}^\alpha, \overline{X}^\alpha]$.

Define a map

$$\overline{d} : L(R) \times L(R) \to R$$

by

$$\overline{d}(X,Y) = \sup_{0 \leqslant \alpha \leqslant 1} d(X^\alpha, Y^\alpha)$$

For $X, Y \in L(R)$ define $X \leqslant Y$ if and only if $\underline{X}^\alpha \leqslant \underline{Y}^\alpha$ and $\overline{X}^\alpha \leqslant \overline{Y}^\alpha$ for each $\alpha \in [0,1]$.

It is known that $L(R)$ is a complete metric space with the metric $\overline{d}$ (see [11]).

Now we define the statistical convergence and λ-statistical convergence of sequences of fuzzy numbers and give an example which compares them.

Definition 2.1 [10]. A sequence $X = (X_k)$ of fuzzy numbers is said to be statistically convergent to the fuzzy number X_0, written as $\text{st} - \lim_k X_k = X_0$, if for every $\varepsilon > 0$,

$$\lim_n \frac{1}{n} |\{k \in N : \overline{d}(X_k, X_0) \geqslant \varepsilon\}| = 0$$

Definition 2.2 ([12]). Let $I_n = [n - \lambda_n + 1, n]$, $\lambda = (\lambda_n)$ be a non-decreasing sequence of positive numbers tending to ∞, $\lambda_{n+1} \leqslant \lambda_n + 1$ with $\lambda_1 = 1$ and $X = (X_k)$ be a sequence of fuzzy numbers. A sequence $X = (X_k)$ of fuzzy numbers is said to be λ-statistically convergent or $s\lambda$-convergent to fuzzy numbers X_0, written as $s\lambda - \lim X_k = X_0$ if for every $\varepsilon > 0$

$$\lim_n \frac{1}{\lambda_n} |\{k \in I_n : \overline{d}(X_k, X_0) \geqslant \varepsilon\}| = 0$$