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A novel approach to fuzzy rough sets based on a fuzzy covering ☆

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Abstract

This paper proposes an approach to fuzzy rough sets in the framework of lattice theory. The new model for fuzzy rough sets is based on the concepts of both fuzzy covering and binary fuzzy logical operators (fuzzy conjunction and fuzzy implication). The conjunction and implication are connected by using the complete lattice-based adjunction theory. With this theory, fuzzy rough approximation operators are generalized and fundamental properties of these operators are investigated. Particularly, comparative studies of the generalized fuzzy rough sets to the classical fuzzy rough sets and Pawlak rough set are carried out. It is shown that the generalized fuzzy rough sets are an extension of the classical fuzzy rough sets as well as a fuzzification of the Pawlak rough set within the framework of complete lattices. A link between the generalized fuzzy rough approximation operators and fundamental morphological operators is presented in a translation-invariant additive group.

Keywords: Fuzzy logic; Conjunction; Implication; Fuzzy covering; Complete lattice; Adjunction; Rough sets; Fuzzy rough approximations; Morphological operators

1. Introduction

A rough set is a set-theory-based technique to handle data with granular structures by using two sets called the rough lower approximation and the rough upper approximation to approximate an object. By using this technique, knowledge hidden in information systems may be unraveled and expressed in the form of decision rules. The classical definition of a rough set was introduced by Pawlak with reference to an equivalence relation (a binary relation with reflexivity, symmetry and transitivity) [28,30,44]. From both theoretical and

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practical viewpoints, the equivalence relation is a very stringent condition that may limit applications of rough sets. Various extensions of the Pawlak rough set were therefore developed from an equivalence relation to a more general mathematical concept, e.g. a similarity relation (a binary relation with reflexivity and symmetry), a covering, or a neighborhood system from topological space [34,39,45,46].

Rough set theory has been successfully used for describing dependency among attributes, evaluating the significance of attributes, and dealing with inconsistent data in knowledge and data analysis. It has provided practical solutions to a number of problems in the information sciences, such as data mining, knowledge discovery, and intelligent control [27,30,33].

In rough set theory, it is assumed that information systems contain only crisp data, and every attribute of an object has a precise and unique value. In most circumstances with data analysis, uncertainty is incorporated into databases. For example, decisions in many knowledge-intensive applications usually involve various forms of uncertainty. The values of attributes in databases may possibly be symbolic or real-valued. In natural language, numerous quantifiers, e.g., many, few, some, almost, are often used for conveying vague information [9,41]. Zadeh [43] carried out a comprehensive investigation on the generalized theory of uncertainty. In this theory, uncertainty is linked to information through the concept of granular structures, and information is represented as a generalized constraint that is drawn from fuzzy set theory and fuzzy logic [42]. Actually, both the concepts of rough sets and fuzzy sets are related but distinct and complementary to each other [2,15,29,38,44]. Many researchers put them together by fuzzifying the underlying rough set to handle data with a fuzzy nature.

Dubois and Prade [17] first introduced the concept of fuzzy rough sets and constructed a pair of fuzzy rough approximations of a fuzzy set by using the notions of the greatest t-norm (min), its dual t-conorm (max), and a fuzzy similarity relation (a fuzzy relation with reflexivity, symmetry and transitivity). Morsi and Yakout [25] developed a generalized definition of fuzzy rough sets by using a lower semi-continuous t-norm $*$, its R-implication, and a fuzzy $*$-similarity relation (a reflexive, symmetric and $*$-transitive fuzzy relation). Axiomatic characterization of the fuzzy rough approximation operators has been studied. Radzikowska and Kerre [31] also presented a general approach to fuzzy rough sets with reference to a t-norm, a special fuzzy implication, and a fuzzy similarity relation. They defined and studied three classes of fuzzy rough sets by taking into account three classes of well-known implications (R-, v- and QL-implications). Other fuzzifications of rough sets have been investigated [1,4,7,21,23,24,36,37]. Fernández Salido and Murakami [18], and Yeung et al. [41] carried out a comparative investigation on those approaches, all of which have been established on the algebraic structure of the unit interval $[0,1]$. It is interesting to extend this structure to a much wider class of mathematical objects, complete lattices.

In this paper, we fuzzify the general rough approximation operators and present a new approach to fuzzy rough sets through the use of techniques provided by lattice theory. The new fuzzy rough approximation operators are established based on a fuzzy covering, a binary fuzzy conjunction logical operator with lower semicontinuity in its second argument, and the adjunctional implication operator of the conjunction. The algebraic properties of the fuzzy rough approximation operators show that the proposed fuzzy rough sets are generalized from the fuzzy rough sets based on possibility theory [14,16,17] and fuzzy inclusions [4,6,23,31,32,35]. A link between the generalized fuzzy rough sets and grey-scale mathematical morphology [3,13,19] is exhibited in a translation-invariant additive domain, and this link may extend the applications of fuzzy rough sets.

The remainder of this paper is organized as follows. A generalized definition of rough sets based on a general covering is presented in Section 2. In Section 3, some essential concepts from fuzzy logic and lattice theory are introduced. The relationship between fuzzy coverings and fuzzy relations is investigated. In Section 4, a new definition of fuzzy rough sets is proposed, and fundamental properties of the fuzzy rough approximation operators are explored. Links between the proposed fuzzy rough sets and classical fuzzy rough sets are discussed in Section 5. Concluding remarks are given in Section 6.

2. Generalized rough sets

2.1. Definition of a Pawlak rough set

Suppose that E is a non-empty set (the universe of discourse) and R is an equivalence relation on E; the pair $(\mathscr{P}(E), R)$ is called a Pawlak rough universe, where $\mathscr{P}(E)$ denotes the powerset of E.