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Solving fuzzy (stochastic) linear programming problems using superiority and inferiority measures

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Abstract

In this paper, the author presents a model to measure the superiority and inferiority of fuzzy numbers/fuzzy stochastic variables. Then, the new measures are used to convert the fuzzy (stochastic) linear program into the corresponding deterministic linear program. Numerical examples are provided to illustrate the effectiveness of the proposed method.

Keywords: Fuzzy random variable; Superiority; Inferiority; Fuzzy linear programming; Fuzzy stochastic linear programming

1. Introduction

The main objective of this paper is to solve fuzzy (stochastic) linear programming problems more efficiently; the author proposes a new method to measure the superiority and inferiority of the triangular fuzzy numbers/fuzzy stochastic variables. Here, the triangular fuzzy numbers/fuzzy random variables are considered because the triangular form is the simplest type of fuzzy numbers/fuzzy random variables. Moreover, we can express and estimate many other types of fuzzy numbers with this simple form of fuzzy number. A triangular fuzzy number gives us the most important information about a fuzzy number: lower and upper bounds of the number and its most possible value. The other types of fuzzy numbers/fuzzy random variables are out of scope of this paper and will be considered in future research works. The new measures are used to evaluate any violation of the constraints to convert the fuzzy (stochastic) linear program into the standard deterministic linear program (LP). In literature, the general direction to handle the challenge of solving fuzzy (stochastic) linear programming problems is to defuzzify and/or derandomize fuzzy numbers/fuzzy stochastic variables [11,6,4,2,10,3,9,7,17,18,16,13,14,12,15,8]. The main difference between the developed methods is the way to defuzzify and/or derandomize fuzzy numbers/fuzzy stochastic variables. The most popular approach to convert the fuzzy linear program (FLP) into the conventional deterministic linear program (LP) is the method of ordering fuzzy numbers such as the area compensation method [5], the expected mid-point of fuzzy numbers [9], the grade of possibility and necessity [1], the signed distance method [3]. Similarly, the fuzzy stochastic

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linear program (FSLP) has also been handled by defuzzifying and derandomizing fuzzy random variables in the sequential manner [17,18,16,8] or in the simultaneous manner [13,14,21,12,20,15,19]. In the sequential approaches, the defuzzifying process is performed first. Then the derandomizing process is done later. The defuzzifying process often utilizes ranking operations or the discretizing process of fuzzy sets via α-levels to defuzzify partly/completely fuzzy stochastic variables. The derandomizing process implements traditional stochastic programming techniques, for example, the chance constrained programming approach or the two-stage programming approach (referred by [18]). The main disadvantage of the sequential method is to create a large number of additional constraints and variables. In the simultaneous approaches, both defuzzifying and derandomizing processes have been performed at the same time by calculating the expected value of fuzzy random variables. Although the obtained deterministic linear program is quite simple, the computation process of the expected value is quite complicated and time consuming. In this paper, the proposed method still follows the sequential method but the new measures could reduce significantly number of additional constraints and variables in the obtained LP. In addition, the main point of the proposed approach is completely different with traditional defuzzifying approach using ranking method; we propose two new measures of inferiority and superiority without ranking fuzzy (stochastic) variables/numbers. These characteristics could be the main reason to simplify our solution process and make the conversion execute more efficiently. One more difference between the proposed approach and traditional ones is that the traditional methods defuzzify fuzzy numbers/fuzzy random variables by using the absolute values between the converted points of fuzzy numbers while the proposed approach uses the relative relationship between fuzzy numbers/fuzzy stochastic variables via the superiority/inferiority degrees.

In fuzzy stochastic linear programming problem, fuzziness and randomness could happen at the same time because the nature of two uncertainties is different. Fuzzy number represents the incomplete, imprecise information. The stochastic variable represents randomness or chance of events. Fuzzy numbers can vary randomly in real life. For example, the estimation of tolerance of machining products can be estimated as a fuzzy number. These values can vary from time to time, cycle-by-cycle in the production lot. Thus, tolerance values could be modeled as fuzzy random variables. In real-life, fuzzy stochastic linear programming arises in several situations. The parameters of linear programs such as the right-hand-sides (RHSs) and coefficients of the objective and constraints could be fuzzy random variables due to the fact that they depend on many factors. Thus, it is difficult to determine exactly the values of these parameters. Moreover, the factors, which are fluctuating due to uncertain environment, could make these parameters vary. These circumstances often happen in long-term planning, development strategies [17] engineering design [25], and financial modeling [26], in which the described conditions (objectives, constraints, coefficients) cannot be determined precisely and certainly. An illustrated example of fuzzy stochastic linear programming could be the case of the production planning problem. Consider the objective of minimizing total cost. This objective can be expressed as a fuzzy stochastic variable because total cost includes cost of inventory holding, materials, and operation. Production output depends on process parameters (for example, cutting speeds, feed rates) and machine running time. However, machine running time is fluctuating and hard to estimate precisely. In addition, available resources, demand, and constraints' coefficients can also be modeled as fuzzy random variables because the vague perceptions with hard statistical data in several environmental conditions such as seasonal factors, market prices, and suppliers which contribute to constraint parameters. Another example is the case of preventive maintenance. Equipment breaks down from time to time, causing losses in production output. To reduce the number of breakdowns, preventive maintenance can be made. Preventive works include inspection, repair, and/or replacement components if necessary. These works cost money in terms of materials, wages, and loss of production due to downtime for preventive works. The length of the downtime is also uncertain due to the complexity of inspection, repair and/or replacement jobs and the skills of maintenance staff. The problem is to determine the preventive frequency such that total downtime, which includes downtime due to breakdowns and downtime due to preventive maintenance, is minimized subject to their associated costs would not exceed the available budget. Here, the running time of the machine is also uncertain. Therefore, we would rather consider these times and their associated costs as fuzzy random variables. These examples motivate the author to propose a new model for solving fuzzy stochastic linear programming problems. One more example of such applications will be discussed in Section 5. The next section presents some important concepts of fuzzy random variables, which will be used as a foundation of the proposed method. Then, the superiority and