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Filter-based resolution principle for lattice-valued propositional logic LP(X) ☆

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Abstract

As one of most powerful approaches in automated reasoning, resolution principle has been introduced to non-classical logics, such as many-valued logic. However, most of the existing works are limited to the chain-type truth-value fields. Lattice-valued logic is a kind of important non-classical logic, which can be applied to describe and handle incomparability by the incomparable elements in its truth-value field. In this paper, a filter-based resolution principle for the lattice-valued propositional logic LP(X) based on lattice implication algebra is presented, where filter of the truth-value field being a lattice implication algebra is taken as the criterion for measuring the satisfiability of a lattice-valued logical formula. The notions and properties of lattice implication algebra, filter of lattice implication algebra, and the lattice-valued propositional logic LP(X) are given firstly. The definitions and structures of two kinds of lattice-valued logical formulae, i.e., the simple generalized clauses and complex generalized clauses, are presented then. Finally, the filter-based resolution principle is given and after that the soundness theorem and weak completeness theorems for the presented approach are proved.

Keywords: Resolution principle; Filter; Lattice-valued logic; Automated reasoning; Simple generalized clause; Complex generalized clause

1. Introduction

Nowadays, considerable attention has been paid to the theories and methods of automated reasoning. Among a variety of approaches for automated reasoning, the resolution principle introduced by Robinson

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[56] is a major one for its simplicity, soundness, and completeness. By the resolution principle, that to prove a theorem is realized by checking whether the equivalent clause set is satisfiable or not. If the clause set is unsatisfiable, then the theorem is proved in the sense of resolution principle. Since Robinson presented the resolution principle, a great number of resolution strategies have been proposed, studied and applied to various application areas such as mathematics, biology, engineering technologies, and so on [17,32,57]. For example, some well-known resolution strategies include the unit-preference strategy [69], the set-of-support strategy [70], the hyper-resolution strategy [32], the semantical resolution strategy [6], the linear resolution strategy [33], the input resolution strategy [6], the non-clausal resolution strategy [44], the T-resolution method in [53], the model elimination strategy for T-resolution method [14], and the hyper-linking resolution strategy [28], etc.

As one of the most important approaches, the resolution principle is affected by the underlying logics. To our best knowledge, most of the existing resolution-based automated reasoning methods are limited to the Kleene's implication operator, i.e., $p \Rightarrow q = \neg p \vee q$, and the chain-type truth-value fields. However, it is known that the classical logic is designed much for an idealized world rather than the real world we are all living in, and is hard to describe and handle uncertainties effectively. To propose appropriate models for processing various uncertainties, more and more researchers paid their attentions to non-classical logics with different application and philosophy backgrounds. Łukasiewicz [37], Post [54], Gottwald [19], Hájek [22], Novák [48], Pavelka [52], etc., have presented their distinguished works in this field. With the rapid progress in theory and successful application in practices of non-classical logics, study of resolution principle based on non-classical logics has been one of hot spots in automated reasoning. Hitherto, a number of automated reasoning tools, such as Barcelona [46], CLIN-S [7], DISCOUNT [8], Isabelle [47], LINUS [29], and OTTER [40], have been designed and applied to mathematical theorem proving, hardware designing, software verification, etc.

In the framework of non-classical logic, perhaps it is Morgan [41] who presented the earliest work on many-valued resolution-based deduction system. Afterwards, Orlowska [49–51] proposed an automated reasoning theory and applied it to an ω^{+}-valued logic as well as the Post's finitely valued logic. Schmitt [60] discussed a resolution-based proving system for a certain 3-valued logic. Hähnle [20], Murray and Rosenthal [45] developed the semantic tableau method for many-valued logic. Baaz et al., discussed the complexity of resolution proving [2] and investigated the resolution-based automated theorem proving for finite-valued quantificational first-order logic [3]. Formisano and Polocriti [14] studied the T-resolution strategy. Gabbay and Reyle [16] investigated the labelled resolution for classical and non-classical logic. Cárdenas Viedma et al. [5], proposed a resolution principle in order to solve queries in a fuzzy temporal constraint logic, and proved that the new resolution principle is a generalization of the one proposed by Dubois et al., in [12] for a possibilistic logic with fuzzy predicates . In [1], Aguilera et al., presented the TAS (abbreviation for a Spanish terminology) method as a new framework for generating non-clausal automated reasoning provers. Kim et al., introduced the antonym-based fuzzy hyper-resolution and proved its completeness theorem in [26]. Smutná-Hliněná et al., discussed a sound resolution deduction for many-valued logic on the unit-interval $[0,1]$ in [61]. Other researchers, such as Chang and Lee [6], Mukaidono [42,43], Yager [81], Dubois and Prade [10–12], Liu and Xiao [32], Kifer [24,25], Stachniak [63], Buro and Büning [4], Lu and Murray [34–36], Dixon [9], Fermüller and Leitsch [13], Policriti and Schwartz [53], Schmidt [59], have proposed their instructive results for fuzzy logic, possibility logic, modal logic, etc.

As an kind of important non-classical logics, lattice-valued logic provides facilities to describe and deal with information or knowledge with incomparability. Hence the study of automated reasoning for lattice-valued logic is of great significance. To our best knowledge, the works about automated reasoning for lattice-valued logic we can refer to are presented by Hähnle, Salzer, Lu, Murray and Rosenthal, Sofronie-Stokkermans and Xu. Hähnle [21] provided tools for distribution quantifiers based on finite distributive lattice. Salzer [58] investigated operators and quantifiers based on semi-lattices. Lu, Murray and Rosenthal [36] extended the signed resolution to dually relatively-pseudo-complemented lattices. Sofronie-Stokkermans [64] presented the method based on a finite distributive lattice with operators by the Priestly dual of the underlying truth-value algebra, and proved the completeness theorem.

To establish an alternative logic for knowledge representation and reasoning, Xu [71] proposed a logical algebra—lattice implication algebra—in 1993 by combining algebraic lattice and implication algebra. In a lattice implication algebra, the lattice is defined to describe uncertainties, especially for the incomparability,