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On the position of intuitionistic fuzzy set theory in the framework of theories modelling imprecision

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Abstract

Intuitionistic fuzzy sets [K.T. Atanassov, Intuitionistic fuzzy sets, VII ITKR's Session, Sofia (deposed in Central Science-Technical Library of Bulgarian Academy of Science, 1697/84), 1983 (in Bulgarian)] are an extension of fuzzy set theory in which not only a membership degree is given, but also a non-membership degree, which is more or less independent. Considering the increasing interest in intuitionistic fuzzy sets, it is useful to determine the position of intuitionistic fuzzy set theory in the framework of the different theories modelling imprecision. In this paper we discuss the mathematical relationship between intuitionistic fuzzy sets and other models of imprecision.

Keywords: Links between different models; Fuzzy set; Interval-valued fuzzy set; Intuitionistic fuzzy set; *L*-fuzzy set; Intuitionistic *L*-fuzzy set; Interval-valued intuitionistic fuzzy set; Probabilistic set; Soft set; Type 2 fuzzy set

1. Introduction

Intuitionistic fuzzy sets are defined by Atanassov in 1983 [1] and form an extension of fuzzy sets. While fuzzy sets only give a membership degree to each element of the universe, and the non-membership degree equals one minus the membership degree, in intuitionistic fuzzy set theory the two degrees are more or less independent, the only constraint is that the sum of the two degrees must not exceed one. Considering the increasing world-wide interest in intuitionistic fuzzy sets – after 20 years of existence already more than 600 papers have been published on intuitionistic fuzzy sets (an overview can be found in [13]), a book has been written [3], since 1997 a conference devoted to intuitionistic fuzzy sets is yearly organized as well as several special sessions on international conferences, it is useful to investigate the position of intuitionistic fuzzy set theory in the framework of the different theories modelling imprecision.

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2. The different models of imprecision

2.1. Fuzzy sets

Definition 2.1 [15]. A fuzzy set A in a universe U is a mapping

$$A : U \to [0,1] : \\ u \mapsto A(u), \quad \forall u \in U.$$

For any $u \in U$, $A(u)$ denotes the membership degree of u in A. The class of fuzzy sets in U is denoted by $\mathscr{F}(U)$.

A fuzzy set A in U is said to be contained in a fuzzy set B in U (notation $A \subseteq B$) if and only if $A(u) \leqslant B(u)$, for all $u \in U$. The union and the intersection of two fuzzy sets A and B in U is given by, for all $u \in U$,

$$A \cup B(u) = \max(A(u), B(u)),$$
$$A \cap B(u) = \min(A(u), B(u)).$$

2.2. Interval-valued fuzzy sets

Instead of assigning a membership degree to each element u of the universe U, an interval-valued fuzzy set A gives for each $u \in U$ a subinterval of $[0,1]$ approximating the "real" but unknown membership degree of u in A.

Definition 2.2 [14]. An interval-valued fuzzy set A in a universe U is a mapping

$$A : U \to \mathrm{Int}([0,1]) : \\ u \mapsto [\underline{A}(u), \overline{A}(u)] \subseteq [0,1], \quad \forall u \in U,$$

where $\mathrm{Int}([0,1])$ is the set of all closed subintervals of $[0,1]$. The class of interval-valued fuzzy sets in a universe U is denoted by $\mathscr{IVFS}(U)$.

The union and the intersection of two interval-valued fuzzy sets A and B in U is given by, for all $u \in U$,

$$A \cup B(u) = [\max(\underline{A}(u), \underline{B}(u)), \max(\overline{A}(u), \overline{B}(u))],$$
$$A \cap B(u) = [\min(\underline{A}(u), \underline{B}(u)), \min(\overline{A}(u), \overline{B}(u))].$$

2.3. Intuitionistic fuzzy sets

Intuitionistic fuzzy sets constitute a generalisation of the notion of a fuzzy set and were introduced by Atanassov in 1983 in [1]. While fuzzy sets give the degree of membership of an element in a given set, intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership, which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than 1.

Definition 2.3 ([1,2]). An intuitionistic fuzzy set A in a universe U is an object of the form

$$A = \{(u, \mu_A(u), \nu_A(u)) \mid u \in U\},$$

where, for all $u \in U$, $\mu_A(u) \in [0,1]$ and $\nu_A(u) \in [0,1]$ are called the membership degree and the non-membership degree, respectively, of u in A, and furthermore satisfy $\mu_A(u) + \nu_A(u) \leqslant 1$. The class of intuitionistic fuzzy sets in U is denoted by $\mathscr{IFS}(U)$.

An intuitionistic fuzzy set A is said to be contained in an intuitionistic fuzzy set B (notation $A \subseteq B$) if and only if $\mu_A(u) \leqslant \mu_B(u)$ and $\nu_A(u) \geqslant \nu_B(u)$, for all $u \in U$.

The union and the intersection of two intuitionistic fuzzy sets A and B in U is given by:

$$A \cup B = \{(u, \max(\mu_A(u), \mu_B(u)), \min(\nu_A(u), \nu_B(u))) \mid u \in U\},$$
$$A \cap B = \{(u, \min(\mu_A(u), \mu_B(u)), \max(\nu_A(u), \nu_B(u))) \mid u \in U\}.$$