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Feedback stabilization of dissipative impulsive dynamical systems ☆

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Abstract

In this paper, we derive conditions under which a dissipative impulsive dynamical system is asymptotically stabilizable by a feedback controller. Specializing the obtained results to the case of dissipative linear impulsive dynamical systems with the quadratic supply rate, we establish the corresponding sufficient conditions. Finally, simulation results are given to demonstrate the effectiveness of our results.

Keywords: Impulsive dynamical system; Dissipativity; Quadratic supply rate; Storage function; Feedback stabilization

1. Introduction

The feedback stabilization of nonlinear systems is an active research area in nonlinear control systems from 1960s. Feedback stabilization is also an important step in achieving additional control objectives, e.g., asymptotic tracking, disturbance attenuation, artificial intelligence, etc. Many results on feedback stabilization are now available in the literature.

In many engineering problems, stability issues are often linked to the theory of feedback stabilization, and dissipative systems (a dissipative system is the one which postulates that the energy dissipated inside a dynamical system is less than the energy supplied from external source). In the literature of nonlinear control, dissipativity concept was initially introduced by Willems in his seminal two-part papers [29,30] in terms of an inequality involving a storage function and a supply rate. The extension of the work of Willems to the case of affine nonlinear systems was carried out by Hill and Moylan [14,15] and references cited therein. Byrnes and Isidori started to study the dissipativity and stabilization of nonlinear continuous systems based on geometric nonlinear system theory in [4,5]. Recently, researchers have extended the notions of classical

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dissipativity theory using generalized storage functions and supply rates for hybrid dynamical systems. For example, Haddad et al. have developed dissipativity concepts for nonlinear impulsive dynamical systems and left-continuous dynamical systems in [9–11] and nonnegative time-delay systems [8,12]. Hill and Zhao have established dissipativity theory for switched systems in [32] and references cited therein. Liu et al. have studied the robust dissipativity problem for impulsive dynamical systems [24], etc.

Feedback stabilization and dissipativity theory as well as the connected Lyapunov stability theory have been studied for dynamical systems possessing continuous motions. However, there are many real world systems and natural processes which display some kind of dynamic behavior in a style of both continuous and discrete characteristics. For instance, many evolutionary processes, particularly some biological systems such as biological neural networks and bursting rhythm models in pathology, as well as optimal control models in economics, frequency-modulated signal processing systems, and flying object motions, and the like, are characterized by abrupt changes of in the state at certain time instances. This is the familiar impulsive phenomenon and the corresponding systems are called impulsive dynamical systems [2,16,17,20–25]. Recently, researchers have also studied the stability and feedback control problems for other discontinuous hybrid dynamical systems, see [1,6,13,26–28,31] and references cited therein. But the results obtained do not include the feedback stabilization of dissipative impulsive dynamical systems.

The traditional methods used in the study of feedback stabilization of dissipative nonlinear continuous systems are those based on the LaSalle invariance principle [18,19]. Although a generalization of the LaSalle invariance principle for impulsive dynamical systems (even left-continuous) has been established in the literature [7], it is difficult to be used to analyze the feedback stabilization of dissipative nonlinear impulsive dynamical systems because solutions of impulsive dynamical systems are no longer continuous.

In this paper, the stability results for general impulsive dynamical systems obtained in [25], instead of the LaSalle invariance principle, are used to derive the conditions under which a dissipative impulsive dynamical system is asymptotically stabilizable by an output or state feedback controller. We then draw further conclusions by specializing the obtained results to the case of dissipative linear impulsive dynamical systems with a quadratic supply rate. Finally, we present numerical simulation studies to illustrate our results.

2. Preliminaries

In the sequel, let $R^+ = [0, +\infty)$, $\mathrm{N} = \{0, 1, 2,\ldots\}$, and let $U_c \subset R^{m_c}$ and $U_d \subset R^{m_d}$ be compact subsets. A matrix $P > 0$ (respectively, $\geqslant 0$) means that P is a positive (respectively, positive-semi) definite matrix. Let $C^r \triangleq C^r(R^n)$, where $r \in N$ and $n \in N$ are given, be the space of functions such that their partial derivatives up to and including the rth order are continuous on R^n. In particular, C^0 denotes the space of all continuous functions on R^n. Let K be the class of functions ϕ: $R^+ \to R^+$, which is continuous, strictly increasing and $\phi(0) = 0$, K_0 the class of continuous functions ψ: $R^+ \to R^+$ such that $\psi(s) = 0$ if and only if $s = 0$, and PC the class of functions p: $R^+ \to R^+$, where p is continuous everywhere except at t_k, $k \in N$, for which p is left continuous and the right limit $p(t_k^+)$ exists. Let $S_\rho = \{x \in R^n{:}\|x\| \leqslant \rho\}$.

Consider an impulsive dynamical system of the form:

$$\begin{cases} \dot{x}(t) = f_c(x(t)) + g_c(x(t))u_c(t), & t \neq t_k, \\ \Delta x(t) = f_d(x(t^-)) + g_d(x(t^-))u_d(t), & t = t_k, \\ y_c(t) = h_c(x(t)) + J_c(x(t))u_c(t), & t \neq t_k, \\ y_d(t) = h_d(x(t^-)) + J_d(x(t^-))u_d(t), & t = t_k, \end{cases} \tag{1}$$

where $x(t_0) = x_0$; $x(t) \in R^n, u_c(t) \in U_c, y_c(t) \in R^{l_c}$, for $t \in R^+$; $\Delta x(t_k) = x(t_k^+) - x(t_k^-), u_d(t_k) \in U_d, y_d(t_k) \in R^{l_d}$, for $k \in N$; $f_c : R^n \to R^n, g_c : R^n \to R^{n\times m_c}$ are Lipschitz continuous and satisfy $f_c(0) = 0$; $f_d : R^n \to R^n$, $g_d : R^n \to R^{n\times m_d}$ are continuous and satisfy $f_d(0) = 0; h_c : R^n \to R^{l_c}$ satisfying $h_c(0) = 0; J_c : R^n \to R^{l_c\times m_c}$, $h_d : R^n \to R^{l_d}$ satisfying $h_d(0) = 0$; and $J_d : R^n \to R^{l_d\times m_d}$.

We assume that the impulsive time instances satisfy $0 < t_0 < t_1 < t_2 < \cdots < t_k < \cdots$, with $t_k \to \infty$ as $k \to \infty$ and that $x(t)$ is left-continuous at t_k, $k \in N$, i.e. $x(t_k^-) = x(t_k)$.

Let U be the class of all admissible inputs consisting of all continuous functions $u_c(t) \in U_c$ $(t \geqslant 0)$, and all vectors $u_d(t_k) \in U_d$ $(k \in N)$. We assume that $U = (U_c, U_d)$ with $(0,0) \in U$.