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On λ-statistical convergence of difference sequences of fuzzy numbers

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Abstract

In this paper the concept of strongly $\Delta^2_{\lambda_p}$-Cesàro summability of a sequence of fuzzy numbers is introduced. Also some inclusion relations between the set of strongly $\Delta^2_{\lambda_p}$-Cesàro convergent and Δ^2_{λ}-statistically convergent sequences of fuzzy numbers are given.

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1. Introduction

The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh [18] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy orderings, fuzzy measures of fuzzy events, fuzzy

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mathematical programming. Matloka [10] introduced bounded and convergent sequences of fuzzy numbers and studied their some properties. Matloka [10] also has shown that every convergent sequence of fuzzy numbers is bounded. Later on sequences of fuzzy numbers have been discussed by Nanda [12], Nuray [14], Kwon [9], Savaş [15], Wu and Wang [17], Bilgin [3], Başarır and Mursaleen [2,11], Aytar [1], Fang and Huang [5], and many others.

The notion of statistical convergence was introduced by Fast [6] and Schoenberg [16] independently. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory, number theory. Later on it was further investigated from the sequence space point of view and linked with summability theory by Fridy [7], Kwon [9], Nuray [14], Savaş [15] and many others. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. Statistical convergence and its generalizations are also connected with subsets of the Stone–Čech compactification of the natural numbers. Moreover, statistical convergence is closely related to the concept of convergence in probability. The notion depends on the density of subsets of the set $\mathbb{N}$ of natural numbers.

The natural density of a set A of positive integers is defined by

$$\delta(A) = \lim_n \frac{1}{n}|\{k \leqslant n : k \in A\}|,$$

where $|\{k \leqslant n : k \in A\}|$ denotes the number of elements of $A \subseteq \mathbb{N}$ not exceeding n [13]. It is clear that any finite subset of $\mathbb{N}$ have zero natural density and $\delta(A^c) = 1 - \delta(A)$. If a property $P(k)$ holds for all $k \in A$ with $\delta(A) = 1$, we say that P holds for almost all k, we abbreviate this by "*a.a.k.*".

A sequence (x_k) is said to be statistically convergent to L if for every $\varepsilon > 0$, $\delta(\{k \in \mathbb{N} : |x_k - L| \geqslant \varepsilon\}) = 0$. In this case we write $S - \lim x_k = L$.

The existing literature on statistical convergence appears to have been restricted to real or complex sequences, but in [9] Kwon, Nuray [14] and Savaş [15] extended the idea to apply to sequences of fuzzy numbers. Also in [3] Bilgin introduced the concept of the Δ-statistical and strong Δ-Cesàro convergence of sequences of fuzzy numbers. In this paper we continue to study the statistical convergence and Cesàro convergence of sequences of fuzzy numbers. We introduce the concepts of strongly $\Delta^2_{\lambda p}$-Cesàro summability and Δ^2_{λ}-statistical convergence of sequences of fuzzy numbers. The obtained results are more general than those of Bilgin [3], Kwon [9] and Savaş [15].

Let $C(\mathbb{R}^n) = \{A \subset \mathbb{R}^n : A \text{ compact and convex}\}$. The space $C(\mathbb{R}^n)$ has linear structure induced by the operations $A + B = \{a + b : a \in A, b \in B\}$ and $\lambda A = \{\lambda a : a \in A\}$ for $A, B \in C(\mathbb{R}^n)$ and $\lambda \in \mathbb{R}$. The Hausdorff distance between A and B of $C(\mathbb{R}^n)$ is defined as