Available online at www.sciencedirect.com
SCIENCE DIRECT®
Information Sciences 176 (2006) 2467–2506
INFORMATION SCIENCES
AN INTERNATIONAL JOURNAL
www.elsevier.com/locate/ins

Balanced fuzzy sets ☆

Władysław Homenda

Faculty of Mathematics and Information Science, Warsaw University of Technology, pl. Politechniki 1, 00-660 Warszawa, Poland

Received 22 August 2004; received in revised form 29 November 2005; accepted 1 December 2005

Abstract

The paper presents a new approach to fuzzy sets and uncertain information based on an observation of asymmetry of classical fuzzy operators. Parallel is drawn between symmetry and negativity of uncertain information. The hypothesis is raised that classical theory of fuzzy sets concentrates the whole negative information in the value 0 of membership function, what makes fuzzy operators asymmetrical. This hypothesis could be seen as a contribution to a broad range discussion on unification of aggregating operators and uncertain information processing rather than an opposition to other approaches. The new approach "spreads" negative information from the point 0 into the interval $[-1,0]$ making scale and operators symmetrical. The balanced counterparts of classical operators are introduced. Relations between classical and balanced operators are discussed and then developed to the hierarchies of balanced operators of higher ranks. The relation between balanced norms, on one hand, and uninorms and nullnorms, on the other, are quite close: balanced norms are related to equivalence classes of some equivalence relation build on linear dependency in the spaces of uninorms and nullnorms. It is worth to stress that this similarity is raised by two entirely different approaches to generalization of fuzzy operators. This observation validates the generalized hierarchy of fuzzy operators to which both approaches converge. The discussion in

☆ This work is supported under State Committee for Scientific Research Grant No. 3T11C00926, years 2004–2007.

E-mail address: homenda@mini.pw.edu.pl

0020-0255/$ - see front matter
doi:10.1016/j.ins.2005.12.003

this paper is aimed at presenting the idea and does not aspire to detailed exploration of all related aspects of uncertainty and information processing.

Keywords: Fuzzy sets; Membership functions; Negative information; Triangular norms; Uninorms and nullnorms; Balanced norms

1. Introduction

The paper presents a new approach to fuzzy sets and uncertain information based on an observation of asymmetry of classical fuzzy operators. Introductory remarks and up to date background of the paper is presented in this section. This presentation includes notes on works related to the merit of this paper as well as fundamental definitions and properties of t-norms, t-conorms, uninorms and nullnorms. Motivation and aspiration to further discussion are outlined in this section. Balanced operators are introduced in Section 2. Definitions and basic properties of balanced t-conorms and balanced t-norms are included. Normal and weak forms of balanced t-norms are considered. Examples of balanced t-conorms and t-norms supplement the section and the balanced counterpart of classical maximum and minimum operators is set up. Section 3 brings up a discussion on similarities between balanced t-conorms and t-norms on one hand, and uninorms and nullnorms, on the other hand. These similarities prompt us to build a broader range of fuzzy operators. The hierarchies of balanced operators are constructed based on iterative operators. Classical t-norms and t-conorms, balanced and t-norms, uninorms and nullnorms are considered as a basis of the hierarchy of balanced operators.

1.1. Preliminaries

A (crisp) set A in a universe X can be defined in the form of its characteristic function $v_A: X \to \{0,1\}$ yielding the value 1 for elements belonging to the set A and the value 0 for elements excluded from the set A. This representation allows for an easy definition of set operations: union, intersection and complement. The *max*, *min* and *complement to* 1 (1−), applied to characteristic functions of respective sets, express the crisp set operations.

A classical fuzzy set A in a universe X can be defined in terms of its membership function $\mu_A: X \to [0,1]$. Membership functions, by analogy to characteristic functions, define fuzzy connectives: union, intersection and complement. The definitions are expressed, as in the case of the crisp sets, by *max*, *min* and *complement to* 1 (1−), cf. [41].

One can observe an asymmetry of the set of values of characteristic function and membership functions: if the state of (certain) inclusion of an element is