Available online at www.sciencedirect.com
SCIENCE DIRECT®
Information Sciences 176 (2006) 1463–1487
INFORMATION SCIENCES
AN INTERNATIONAL JOURNAL
www.elsevier.com/locate/ins

On the use of words and fuzzy sets ☆,☆☆

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Received 23 November 2004; received in revised form 2 March 2005; accepted 4 March 2005

Abstract

This paper only tries to stimulate some reflections, by showing a possible way towards rethinking fuzzy sets from their roots. A rethinking that does not change the view that fuzzy sets are mathematical entities giving extension to predicates. In such a way that, if the predicates are precise, the corresponding new entities are nothing else than classical sets.

What, perhaps, is a key idea is that the use of a predicate organizes, in some way, the universe of discourse. When this organization is a preorder, the extension or L-set, appears once a degree, numerical or not, but consistent with the organization, can be defined.

The ultimate goal of those above mentioned reflections, provided they would be realized in the future, is to extend the current theories of fuzzy sets to wider areas of both language and reasoning. Our objective is to reach a better knowledge of the links between language and its representations for the progress of computing with words and perceptions.

Keywords: Predicates; Gradable predicates; L-sets; Fuzzy sets

☆ To Professor Lotfi A. Zadeh.

☆☆ Paper partially funded by the European Project Omni Paper (IST-2001-32174).

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0020-0255/$ - see front matter
doi:10.1016/j.ins.2005.03.008

1. Introduction

If, in some sense, the 1965 paper of Zadeh on fuzzy sets can be viewed as a pioneering move from Cybernetics to Artificial Intelligence, Mamdani's introduction of (intelligent) fuzzy control in 1975 can be considered to represent, if not the first, at least one of the first Expert Systems. In short, from its beginning fuzzy logic deals with computational intelligence and, in the 40 years after 1965 and in the 30 after 1975, a lot of theoretical results and technological applications flourished in the field of fuzzy logic. A field that, under Zadeh's suggestions and insights, is now changing to the wider one of computing with words and perceptions.

Fuzzy logic not only deals with problems at the technological side of computational intelligence. Since what a fuzzy set does represent is a concrete use of a predicate (or linguistic label), and as Wittgenstein asserted, "the meaning of a word is its use in the language", fuzzy logic also deals with what is known as the *Gordian Knot* of computational intelligence, the problem of meaning. And this is a side of fuzzy logic that, in the way towards computing with words, seems to be of great interest. Words are context-dependent, and if there are uncountable many theories of fuzzy sets it is, because fuzzy logic recognizes that there is no a single way for using the conjunction 'and', the disjunction 'or', and the negation 'not', and that the diverse representations of these linguistic connectives are based on the particular properties of their current meanings, on their use in a given context and for some goal.

Generally speaking, the use of a predicate on a universe of discourse, if gradable, defines a collective in the universe that only in some cases is a classical set. Predicates P do collectivize in some way provided the corresponding statements 'x is P' admit a degree, be it immediately numerical or not.

Currently, in the thinking of some people working in fuzzy logic from almost its beginning, it is sprouting the idea that the time to rethink fuzzy sets and fuzzy logic is coming. Such rethinking is not only viewed to push ahead the knowledge of (linguistic) imprecision, and the corresponding uncertainty, but to give a better support to the more complex applications that are foreseeable in a not too long future.

What this paper tries to offer from a theoretical point of view, are just some hints in the direction of extending the theories of fuzzy sets to face broader areas of language than those they can currently deal with. It is not to be forgotten that, as Searle said, "Speaking a language is engaging in a rule-governed form of behavior".

Although the paper will refer only to mathematical models, before beginning with them it is important to declare, like in the customs, that if mathematics are really important for the understanding of the phenomena linked to imprecision, they are nothing more and nothing less than a basic tool for such a goal. What really matters is imprecision and, of course, mathematical models can help both