Available online at www.sciencedirect.com
SCIENCE DIRECT®
Information Sciences 176 (2006) 887–899
INFORMATION SCIENCES
AN INTERNATIONAL JOURNAL
www.elsevier.com/locate/ins

The minimization of axiom sets characterizing generalized approximation operators

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Received 31 August 2004; accepted 18 January 2005

Abstract

In the axiomatic approach of rough set theory, rough approximation operators are characterized by a set of axioms that guarantees the existence of certain types of binary relations reproducing the operators. Thus axiomatic characterization of rough approximation operators is an important aspect in the study of rough set theory. In this paper, the independence of axioms of generalized crisp approximation operators is investigated, and their minimal sets of axioms are presented.

Keywords: Approximation operators; Axioms; Rough sets

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doi:10.1016/j.ins.2005.01.012

1. Introduction

Rough set theory is a generalization of the classical set theory for modelling systems with uncertain or incomplete information, and it has recently aroused much interest in both theory and applications. For example, it may be used to unravel knowledge hidden in information systems and express the knowledge in the form of decision rules.

There are two approaches to the development of rough set theory: constructive and axiomatic. In the constructive approach, primitive notions include binary relations on the universe, partitions of the universe, neighborhood systems and Boolean algebras, and based on these notions the lower and upper approximation operators are constructed [6,8,9,14–16,18,24,26,28–33,35]. On the other hand, the axiomatic approach takes the lower and upper approximation operators as primitive notions, and a set of axioms is used to characterize the approximation operators produced using the constructive approach. In terms of axiomatic approach, rough set theory may be interpreted as an extension of the classical set theory with two additional unary operators. The lower and upper approximation operators are related to the necessity (box) and possibility (diamond) operators in modal logic, and the interior and closure operators in topological space [2,3,7,9,10,19,23,29,30,32]. Under this approach, a set of axioms is used to characterize approximation operators that are the same as the ones produced by using constructive approach.

Zakowski [34] studied a set of axioms on approximation operators, and Comer [4,5] investigated axioms on approximation operators in relation to cylindric algebras within the context of Pawlak information systems [13]. Lin and Liu [9] proposed six axioms on a pair of abstract operators on the power set of universe in the framework of topological space, under which there exists an equivalence relation reproducing the lower and upper approximation operators by the constructive approach. Similar result was reported by Wiweger [23]. However, these studies are restricted to Pawlak rough set algebra defined by equivalence relations. Wybraniec–Skardowska [28] examined many axioms on various classes of approximation operators and proposed several constructive methods to generate them. Mordeson [11] investigated the axiomatic characterization of approximation operators defined by covers, and Thiele [19] explored the axiomatic characterization within modal logic. The most important axiomatic studies for crisp rough sets are done by Yao et al. [29,30,32,33], where various crisp rough set algebras are characterized using different sets of axioms. The research of axiomatic approach has also been extended to approximation operators in fuzzy environment [1,10,12,17,20–22,25,27].

However, the abovementioned studies have not solved the important problem of the independence and minimization of the axiom set for approximation operators. This paper attempts to solve this problem for generalized crisp