International Journal of Approximate Reasoning 76 (2016) 63–79

Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

www.elsevier.com/locate/ijar

Dynamic consistency in incomplete information games under ambiguity

Hailin Liu, Wei Xiong *

Institute of Logic and Cognition, Department of Philosophy, Sun Yat-Sen University, China

ARTICLE INFO

Article history:
Received 17 November 2015
Received in revised form 3 March 2016
Accepted 15 April 2016
Available online 22 April 2016

Keywords:
Dynamic consistency
Incomplete information games
Ambiguity
Γ-maximin
Rectangularity

ABSTRACT

We develop a general framework of incomplete information games under ambiguity which extends the traditional framework of Bayesian games to the context of Ellsberg-type ambiguity. We then propose new solution concepts called *ex ante* and *interim* Γ-maximin equilibrium for solving such games. We show that, unlike the standard notion of Bayesian Nash equilibrium, these concepts may lead to rather different recommendations for the same game under ambiguity. This phenomenon is often referred to as *dynamic inconsistency*. Moreover, we characterize the sufficient condition under which dynamic consistency is assured in this generalized framework.

© 2016 Elsevier Inc. All rights reserved.

1. Introduction

Many economic situations of interest involve interactive decision making under uncertainty in which the participants have only partial information about the payoff relevant aspects of the circumstance. In his seminal contribution, Harsanyi [16] proposes to model such strategic situations by introducing incomplete information games. The basic picture of Harsanyi's game model goes like this. There is a fundamental state space that includes all the possibilities of the players' relevant characteristics. And the players possess different private information about the situation in which they interact. Nevertheless, it is assumed that all the players in the game employ a *single*, *common* prior to represent their beliefs about the states before having received any private information. Upon the arrival of new information about the states, each player is required to revise her initial belief by applying Bayes' rule.[1] As a final step, each player is asked to choose a strategy that maximizes her expected utility with respect to this updated probability distribution and her belief about the other players' strategy choices, which is called the player's *best response*. The requirement of being best response to each other's choice thus gives rise to the most commonly used solution concept for Bayesian games known as *Bayesian Nash equilibrium*. Such a framework provides a powerful tool for analyzing strategic situations involving differences of private information among the players, which has a wide range of applications in various domains such as auction, mechanism design and jury voting [12,34].

An important feature of Bayesian Nash equilibrium is that it ensures the so-called *dynamic consistency* property, which has been extensively discussed in the context of sequential decision making [17,39,29,20,30,21]. Roughly speaking, dynamic consistency requires that an optimal contingent plan of action will remain optimal when new information is updated. In the

* Corresponding author.
E-mail addresses: liuhlin3@mail.sysu.edu.cn (H. Liu), hssxwei@mail.sysu.edu.cn (W. Xiong).

[1] Such a game model is commonly known as "Bayesian" game exactly due to its use of Bayes' rule for updating players' beliefs in the light of new information.

http://dx.doi.org/10.1016/j.ijar.2016.04.005
0888-613X/© 2016 Elsevier Inc. All rights reserved.

context of Bayesian games, this implies that, upon the receipt of private information, the players would not want to revise their planned actions that they deem as optimal when evaluated from the *ex ante* perspective. As a matter of fact, within the framework of Bayesian games, there is no need to distinguish between *ex ante* and *interim* Bayesian Nash equilibrium, since it makes no difference whether the players evaluate their strategies using the *ex ante* or *interim* versions of expected utility maximization. Such an equivalence result is not surprising at all, for expected utility model does satisfy dynamic consistency due to the law of iterated expectations. The intuition behind the law of iterated expectation lies in the fact that a (joint) probability distribution can be decomposed into expressions involving its conditional and marginal distribution. As we shall see, this decomposition property plays a critical role in the latter discussion concerning dynamic consistency in the context of non-expected utility.

It is important to note, however, that Harsanyi's theory of Bayesian games relies heavily on the assumption of the existence of a single common prior representing all the players' initial beliefs.[2] This assumption seems extremely implausible in many real world situations. Furthermore, a number of researchers have cast doubt on the adequacy of modelling uncertainty using a single probability distribution in a variety of contexts. Knight [23] raises questions about the suitability of probabilistic characterization of uncertainty in certain situations, which leads to the important distinction between risk and uncertainty. Since the seminal work of Ellsberg [7], many authors have convincingly argued that the representation of uncertainty in terms of a single precise probability function is not only practically unrealistic but also normatively insufficient in many situations [24–26,50]. Motivated by this, a vast literature has developed on extending expected utility theory to accommodate the Ellsberg paradox by modelling uncertainty through a set of probabilities rather than a precise probability distribution [13,38,41,42,50]. In light of these, it is natural to think that the common prior assumption in Bayesian games should be relaxed as well in order to accommodate Ellsberg-type uncertainty, which constitutes the primary motivation of the current work.

In this paper, we take the view that due to limited information a player may not be able to identify a unique probability for representing her belief about the states, which thus should be depicted as a set of probability distributions instead. Based on this view, we present a general model of incomplete information games, which extends the traditional framework of Bayesian games to include Ellsberg-type ambiguity. Following the pioneer work of Kajii and Ui [19], we assume that each player's perception of uncertainty about the states is modelled by a compact set of probability measures, instead of a single common probability distribution. Moreover, we allow for the possibility that different players may have different initial beliefs, which can be represented by uncommon sets of probability distributions. We also adopt the principle of Γ-maximin as the decision rule used by all the players.[3] Similarly to Kajii and Ui's approach, we propose to study equilibrium concepts in which each player chooses the optimal action in the sense of maximizing the minimum expected utility for each realization of her private signal. In this sense, our model constitutes only a minor departure from the standard approach to games with incomplete information.

Nevertheless, our model differs from the model introduced by Kajii and Ui [19] in the following respects. Kajii and Ui assume that each player is endowed with an updating rule for determining their posterior beliefs, whereas we allow for any set of well-defined posterior functions to represent players' updated beliefs. In view of this, our model is a bit more general than the one developed by Kajii and Ui [19]. In addition, it has two advantages over their game model. First, our approach avoids the ongoing controversy concerning the right updating rule under ambiguity, as the theoretical literature on dynamic choices under ambiguity has not yet reached a consensus on which updating rule is most plausible for the modelling of ambiguity through a set of probabilities. Second, it also facilitates the discussion about the relationship between priors and posteriors, which turns out to be crucial for establishing the central result of this paper (see Section 4).

It has already been shown (see for instance Refs. [8,36]) that, non-expected utility decision makers are expected to violate the requirement of dynamic consistency when uncertainty is represented by a set of probability measures. It thus should come as no surprise that the concepts of *ex ante* and *interim Γ-maximin equilibrium* generally do not coincide in the current framework. Indeed, as Kajii and Ui [19] have rightly recognized, an *ex ante* Γ-maximin equilibrium may specify strategies that are not components of any *interim* Γ-maximin equilibrium. In this paper, we reinforce this point by showing an example (see the main example of Section 3) in which the set of *ex ante* Γ-maximin equilibria of the game is disjoint from the one of its *interim* Γ-maximin equilibria. This thus implies that dynamic consistency does not hold in the present framework and these two equilibrium concepts should be treated differently.

In view of violations of dynamic consistency, Kajii and Ui [19] suggest to sidestep this problem by focusing merely on the interim equilibrium concept. It should be remarked that, however, the counterintuitive phenomenon of aversion to cost-free information may occur when the condition of dynamic consistency is violated [48,8]. From a normative point of view, such a behavior seems quite problematic, which is often used as an argument against non-expected utility theories (see for example Ref. [1]).

The main objective of this paper is then to determine under what conditions these two equilibrium concepts become equivalent in the present game-theoretic framework, which in turn guarantees the property of dynamic consistency. The

[2] An alternative but equivalent way of formulating Bayesian games is to assume that its basic primitives are the beliefs of all the players given their private information. As Harsanyi [16] points out, however, such beliefs can be regarded as conditional probability distributions derived from a prior probability measure.

[3] The use of Γ-maximin to decision making under severe uncertainty can be traced back to at least Wald [49] and Hurwicz [18]. Gilboa and Schmeidler [13] supply it with an axiomatic foundation.

Download English Version:

https://daneshyari.com/en/article/396878

Download Persian Version:

https://daneshyari.com/article/396878

Daneshyari.com