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Approximation of centroid end-points and switch points for replacing type reduction algorithms

Syed Moshfeq Salaken [a,*], Abbas Khosravi [a], Saeid Nahavandi [a], Dongrui Wu [b]

[a] *Center for Intelligent Systems Research, Deakin University, Australia*
[b] *Machine Learning Lab, GE Global Research, United States*

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ABSTRACT

Despite several years of research, type reduction (TR) operation in interval type-2 fuzzy logic system (IT2FLS) cannot perform as fast as a type-1 defuzzifier. In particular, widely used Karnik–Mendel (KM) TR algorithm is computationally much more demanding than alternative TR approaches. In this work, a data driven framework is proposed to quickly, yet accurately, estimate the output of the KM TR algorithm using simple regression models. Comprehensive simulation performed in this study shows that the centroid end-points of KM algorithm can be approximated with a mean absolute percentage error as low as 0.4%. Also, switch point prediction accuracy can be as high as 100%. In conjunction with the fact that simple regression model can be trained with data generated using exhaustive defuzzification method, this work shows the potential of proposed method to provide highly accurate, yet extremely fast, TR approximation method. Speed of the proposed method should theoretically outperform all available TR methods while keeping the uncertainty information intact in the process.

1. Introduction

Soon after the introduction of type-2 fuzzy set by Zadeh in 1974 [1], its applicability in different real world application become apparent. Presently, type-2 fuzzy logic system is used with significant success in the field of decision making [2–6], mobile robotics [7,8], control [9,10], prior processing of data [11], forecasting accuracy [12], noise reduction [13], survey processing [14,15], prediction interval construction [16,17], clustering [18], intelligent environment realization [19] and time series forecasting [20,14,21–26], to name a few. This vast field of application generally utilizes high research concentration on computationally simpler version of general type-2 fuzzy sets, interval type-2 fuzzy set (IT2FS) and corresponding interval type-2 fuzzy logic system (IT2FLS) [27–29]. One integral part of IT2FLS is the type reduction (TR) block (see Fig. 1), which generally poses a bottleneck in computation process [30,29,31]. Existing TR algorithms for non-simplified fuzzy sets, which intend to tackle the computational bottleneck using soft algorithmic approach, can be divided into two main classes depending on their approach to preserve the associated uncertainty information during TR operation. Wu termed them as “Enhancements to KM TR algorithm” [30] and “Alternative TR algorithms” [30]. Recently, Greenfield argued that most widely used TR algorithm, Karnik–Mendel algorithm [32], is not the most accurate TR algorithm available [33]. This poses an interesting question. If the exhaustive defuzzification is the most accurate method of TR operation, how should this be

* Corresponding author at: Deakin University, 75 Pigdons Road, Waurn Ponds, Victoria 3216, Australia. Tel.: +61 424262359.
E-mail addresses: syed.salaken@gmail.com, syed.salaken@research.deakin.edu.au (S.M. Salaken), abbas.khosravi@deakin.edu.au (A. Khosravi), saeid.nahavandi@deakin.edu.au (S. Nahavandi), drwu09@gmail.com (D. Wu).

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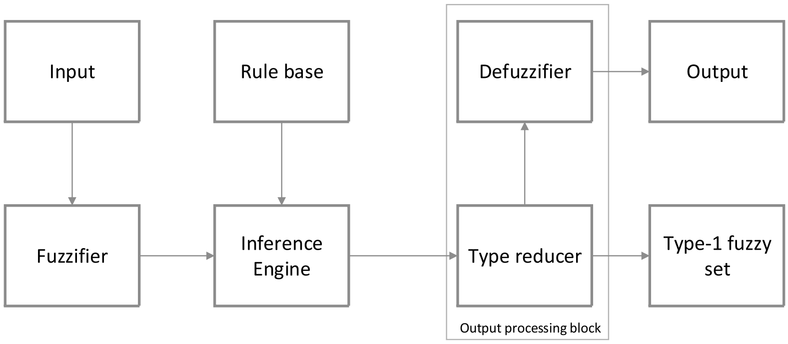

Fig. 1. IT2 fuzzy logic system.

approached while ensuring minimal run-time computational overhead with an acceptable error margin? In this research work, we propose a method to answer this problem. Formally stating, our motivation in this work is to find a systematic approach which will minimize the computational overhead associated with type reduction operation in IT2FLS and T2FLS while ensuring the accuracy of exhaustive defuzzification is preserved.

It is known for a long time that regression can be used for function approximation [34,35]. Here, we show that simple polynomial regression can be used for TR operation with very low error to address the issue mentioned in the above paragraph. Using regression based approach to approximate the underlying function mapping among membership functions (MFs), input and rulebase has a number of advantages including simpler and faster training, high robustness to capture the increased complexities with increasing number of inputs & MFs and the potential of utilizing long established tuning techniques for performance enhancement.

Since the KM type TR algorithms preserve the uncertainty by finding centroid end-points, which is closely intertwined with switch points, this work addresses both of them. Even though either centroid end-point approximation or switch point prediction is sufficient to determine the uncertainty information, we propose regression model for both of them to ensure completeness of the work.

It should be noted that regression model can be trained by either exhaustive defuzzification method, which guarantees most accurate output, or any existing TR algorithm. The essence of this study remains same regardless of the defuzzification method used in data generation stage. For simplicity, Karnik–Mendel TR algorithm is used in this work.

Rest of this paper is structured as follows: Section 2 states KM algorithm in detail and mentions all available TR algorithms for completeness, Section 3 gives an overview of experimental methods, Section 4 describes the data generation methods in detail, Section 5 describes regression modelling technique for both centroid end-points approximation and switch point prediction task, Section 6 discusses the findings and finally, Section 7 concludes the paper.

2. Preliminaries

2.1. Karnik–Mendel algorithm

Karnik and Mendel proposed an algorithm, referred as KM algorithm hereafter, for type reduction in 2001 [32]. Since then KM algorithm has been considered as a benchmark for TR. KM algorithm provides the left and right end-points of centroid which represent the uncertainties associated with a type-2 FLS. We describe the KM algorithm below for the completeness of this work. Mathematical proof of this algorithm can be found in [32].

For the left end of centroid, y_l:

1. Sort rule consequent x_i $(i = 1, 2, 3, \ldots, N)$ in increasing order while keeping the variable name same, but now each element of x_i is positioned in ascending order such that $x_1 \leq x_2 \leq \ldots \leq x_N$.
2. Match the corresponding firing intervals or weights in the order that matches with related original value of rule consequent. In other words, renumber the index of lower and firing intervals, $\underline{w}_i$ and $\overline{w}_i$ respectively, to maintain the original one to one correspondence with rule consequent.
3. Initialize the weight, w_i as follows:

$$w_i = \frac{\underline{w}_i + \overline{w}_i}{2} \qquad \text{where, } i = 1, 2, 3, \ldots, N \tag{1}$$

4. Compute the initial centroid value by

$$y = \frac{\sum_{i=1}^{N} x_i w_i}{\sum_{i=1}^{N} w_i} \tag{2}$$