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A least deviation method for priority derivation in group decision making with incomplete reciprocal preference relations

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ABSTRACT

In this paper, based on the transfer relationship between reciprocal preference relation and multiplicative preference relation, we proposed a least deviation method (LDM) to obtain a priority vector for group decision making (GDM) problems where decision-makers' (DMs') assessments on alternatives are furnished as incomplete reciprocal preference relations with missing values. Relevant theorems are investigated and a convergent iterative algorithm about LDM is developed. Using three numerical examples, the LDM is compared with the other prioritization methods based on two performance evaluation criteria: maximum deviation and maximum absolute deviation. Statistical comparative study, complexity of computation of different algorithms, and comparative analyses are provided to show its advantages over existing approaches.

1. Introduction

Group decision making (GDM) [4,15,21,22,32,43,53] is participatory process in which multiple individuals, often experts, together formulate problems, develop alternatives and eventually select among the alternatives to reach a decision. In order to rank these alternatives, decision makers (DMs) usually express their pairwise comparison information in two formats: a multiplicative preference relation [18,23,41] $A=(a_{ij})_{n\times n}$, where a_{ij} means the relative weight of alternative x_i with respect to x_j and $a_{ij}\in[1/9,9]$, $a_{ij}a_{ji}=1$, and a reciprocal preference relation (also called fuzzy preference relation) [17,25,45,52] $R=(r_{ij})_{n\times n}$, where r_{ij} is an estimate for the relative significance of the alternative x_i and x_j, and $r_{ij}\in(0,1)$, $r_{ij}+r_{ji}=1$. Over past few decades, lots of prioritization methods have been developed to derive priority from a reciprocal preference relation, including goal programing method [9], multi-objective optimization method [12], least-deviation method [52], chi-square method [34] and eigenvector method [35], etc.

Sometimes, however, a DM furnishes his/her judgment on alternatives as a reciprocal preference relation with missing or incomplete entries, because of time pressure, lack of knowledge, and the DM's limited expertise in the specific problem domain [3,11,24,33,40,47]. Up to now, several prioritization methods for incomplete reciprocal preference relations have been proposed, including goal programming model (GPM) [47], least square method (LSM) [14], eigenvector method (EM) [46], logarithmic least squares method (LLSM) [44], normalizing rank aggregation method (NRAM) [38], least variance method

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(LVM) [38]. Xu [49] proposed a procedure for decision making with incomplete reciprocal preference relation based on multiplicative consistency. Xu and Chen [51] developed a simple but practical approach to deriving the ranking of the alternatives from an incomplete reciprocal relation based on additive transitivity. Shen et al. [30] and Xu et al. [39] had pointed out that the corresponding may be unreasonable, and deduced a function between the reciprocal preference relation and priority vector. Herrera-Viedma et al. [16] presented a new model to deal with group decision making (GDM) problems with the incomplete reciprocal preference relations based on the additive-consistency property. The new model is composed of two steps: the estimation of missing preference values and the selection of alternatives.

Each approach has its advantages and disadvantages. Some comparative analysis among the commonly used prioritization methods in the case of multiplicative preference relation can be found in the literature [7,8,13,19,27,29]. Main conclusion is that there is no one method that is superior to the others in all cases and the choice of the prioritization method should be dictated by the objective of the analysis. However, little attention has been paid to the performances of prioritization methods for incomplete reciprocal preference relations. In this paper, based on the transfer relationship between reciprocal preference relation and multiplicative preference relation, a new prioritization method for the priority vector derivation from incomplete reciprocal preference relations is proposed, which is called least deviation method (LDM), and then the proposed LDM is compared with the existing methods regarding two performance evaluation criteria: maximum deviation (MD) and maximum absolute deviation (MAD).

The remainder of the paper is organized as follows. Section 2 provides a review on basic concepts of reciprocal preference relation, incomplete reciprocal preference relation and the transfer relationship between reciprocal preference relation and multiplicative preference relation. In Section 3, the LDM is extended to obtain a priority vector from incomplete reciprocal preference relations based on the transfer relationship, resulting in an iterative algorithm. In Section 4, three examples are examined to show how to apply the proposed LDM and its effectiveness in handling GDM problems. Comparative analyses with existing methods demonstrate its validity and advantages. Statistical comparative study using Wilcoxon signed-rank test is provided in Section 5. Complexity of computation of different algorithms is listed in Section 6. Concluding remarks are furnished in Section 7.

2. Preliminaries

For simplicity, denote $N = \{1, 2, \dots, n\}$, $M = \{1, 2, \dots, m\}$. Let $X = \{x_1, x_2, \dots, x_n\}$ $(n \geq 2)$ be a finite set of alternatives, where x_i denotes the ith alternative, $E = \{e_1, \dots, e_m\}$ be a finite set of experts, where e_k stands for kth expert, $H = (h_1, \dots, h_m)^T$ be the weight vector of experts, where $\sum_{k=1}^{m} h_k = 1$, $h_k \geq 0$ and h_k demonstrates the importance degree of expert e_k in the decision process. A brief description of multiplicative preference relation and reciprocal preference relation is given below [6,9,10].

(1) *Multiplicative preference relation* [28]. The preference information on X is described by a multiplicative preference relation $A \subset X \times X$, $A = (a_{ij})_{n\times n}$, where a_{ij} means the relative weight of alternative x_i with respect to x_j. The measures of a_{ij} are described using a ratio scale $a_{ij} \in \{1/9, 1/8, \dots, 1/2, 1, 2, \dots, 8, 9\}$: $a_{ij} = 1$ denotes indifference between x_i and x_j. $a_{ij} = 9$ denotes that x_i is unanimously preferred to x_j and $a_{ij} \in \{2, 3, \dots, 8\}$ denotes the intermediate evaluations. It is multiplicative reciprocal $a_{ij}a_{ji} = 1$, $a_{ii} = 1$, for all $i, j \in N$.

(2) *Reciprocal preference relation* [25]. The preference information on X is described by a reciprocal preference relation $R \subset X \times X$, $R = (r_{ij})_{n\times n}$, with membership function $\mu_R : X \times X \to [0, 1]$, where $\mu_R(x_i, x_j) = r_{ij}$ denotes the preference degree of alternative x_i over x_j. $r_{ij} = 0.5$ denotes indifference between x_i and x_j. $r_{ij} = 1$ denotes x_i is definitely preferred to x_j. $0 \leq r_{ij} < 0.5$ implies that x_j is preferred to x_i. $0.5 < r_{ij} < 1$ means that x_i is preferred to x_j. It is additive reciprocal, $r_{ij} + r_{ji} = 1$, $r_{ii} = 0.5$, for all $i, j \in N$.

Definition 1. (See [26].) Let $A = (a_{ij})_{n\times n}$ be a multiplicative preference relation, then A is called a consistent multiplicative preference relation, if

$$a_{ij} = a_{ik}a_{kj}, \quad \text{for all } i, j, k \in N \tag{1}$$

It has been found that a consistent multiplicative preference relation can be precisely characterized by a priority vector $W = (w_1, w_2, \dots, w_n)^T$, which satisfies $\sum_{i=1}^{n} w_i = 1$ and $w_i > 0$ for $i \in N$. That is

$$a_{ij} = w_i/w_j \tag{2}$$

Definition 2. (See [31].) Let $R = (r_{ij})_{n\times n}$ be a reciprocal preference relation, then R is called an additive reciprocal preference relation if

$$r_{ij} = r_{ik} - r_{jk} + 0.5, \quad \text{for all } i, j, k \in N \tag{3}$$

Chiclana et al. [6] established a relationship between multiplicative preference relation and reciprocal preference relation as follows: