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A rule-based development of incremental models

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ABSTRACT

In the study, we propose a concept of incremental fuzzy models in which fuzzy rules are aimed at compensating discrepancies resulting because of the use of a certain global yet simple model of general nature (such as e.g., a constant or linear regression). The structure of input data and error discovered through fuzzy clustering is captured in the form of a collection of fuzzy clusters, which helps eliminate (compensate) error produced by the global model. We discuss a detailed architecture of the proposed rule-based model and present its design based on an augmented version of Fuzzy C-Means (FCM). An extended suite of experimental studies offering some comparative analysis is covered as well.

1. Introduction

Rule-based models are composed of collections of rules, which serve as local models describing complex relationships existing in system modeling. Takagi–Sugeno rules [1] assume the well-known form

$$-\text{if } \boldsymbol{x} \text{ is } A_i \text{ then } y \text{ is } f_i(\boldsymbol{x}) \tag{1}$$

where A_i is a fuzzy set of condition of the i-th rule and formed in the input space and $f_i(\boldsymbol{x})$ is a certain local model standing in the conclusion part are commonly encountered in fuzzy modeling.

There is a significant number of design procedures of rule-based models [2–7]. In general, the design of the fuzzy model is carried out in two phases: (i) formation of condition parts of the rules – fuzzy sets built by fuzzy clustering, and (ii) determination of the conclusion parts of the rules; here we are concerned with a parametric optimization and resort the analytical solutions of the ensuing estimation problem. Quite commonly we encounter the local models assuming a simple form being e.g., linear functions. The design process is well documented in the literature [8]. Each of these two design phases comes with various augmentations. In Table 1, we offer several highlights of the visible representatives of the fuzzy models, specify their development strategies and look at the optimization tools supporting the construction of the fuzzy model.

Irrespectively of the diversity of approaches, fuzzy rule-based models share a visible commonality: complex phenomena are modeled *locally* through a series of local models (which are less complex than a single global model). The differences

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Table 1
A collection of design strategies and optimization tools – selected examples.

Construction of condition part of the rules	Construction of conclusion part of the rules	References	Global/Local development
Hybrid algorithm (genetic algorithms and complex method)	Least square error method	[9–12]	Global
Resilient propagation (RPROP) original heuristic search	Standard gradient descent (back propagation) Resilient propagation (RPROP)	[13–15]	Local
Genetic algorithm	Recursive least squares approach	[16–18]	Global
Fuzzy clustering (K-Means, the Gath–Geva algorithm and the Gustafson–Kessel algorithm)	Weighted recursive least squares algorithm with forgetting factor	[19,20]	Local
Fuzzy C-Means (FCM) clustering	Weighted Least Square Estimation (WLSE) method	[21,22]	Local
Fuzzy C-Means (FCM) clustering algorithm modified fuzzy c-regressive model clustering algorithm (NFCRMA)	Orthogonal least square (OLS)	[23,24]	Global
Gravitational search (GSA)-based hyper-plane clustering algorithm (GSHPC)	Orthogonal least square (OLS)	[25–27]	Global
Hard C-means clustering method Genetic algorithms (GAs)	Least square error method	[10,28]	Global
Modified fuzzy c-regression model (FCRM)	Orthogonal least squares	[29–31]	Global
Cluster estimation method	Least mean squares estimation	[32–34]	Global
Back propagation learning rule	Least square method	[1,35,36]	Global
Extended vector quantization	Recursive weighted least-squares approach	[37–40]	Local
A new projection concept	Enhanced recursive least square method	[41–44]	Local
Iterative vector quantization algorithm	Regularized sparsity-constrained-optimization	[45–47]	Local
Subtractive clustering method	Linear least squares estimation	[34,48]	Global

lie in a way in which the rules are formed, how local (typically linear) models are constructed and how aggregation of the rules is completed.

While rule-based models are about local modeling of the overall system, incremental fuzzy models follow a radically different line of thought. We consider a global model coming in a form commonly encountered in the literature (say, a linear regression model) while the rule-based model is constructed to enhance – compensate for some discrepancies of the global model. The individual rules of the incremental model are developed in a way so that the deficiencies of the global model are eliminated or substantially reduced. In this way, the incremental model improves the quality of the global model. Some initial studies in this area were reported in [8]. This incremental rule-based approach to system modeling exhibits some tangible advantages: (i) we rely on the well-known and commonly acceptable model, such as linear regression and offer its improvement, (ii) the methodology of fuzzy rule-based modeling fully applies here so we take advantage of the existing design strategies of fuzzy models, (iii) the two-phase design process is well-motivated.

The objective of the study is to develop a detailed concept of incremental fuzzy models, propose algorithms, and demonstrate the performance of the model with the use of synthetic and publicly available data sets.

The study is structured as follows. In Section 2, we present a structure of the incremental fuzzy model showing how the rules are formed to account for the existing discrepancies of the global model. In Section 3, we discussed augmented fuzzy clustering (extended version of the Fuzzy C-Means), which allows to articulate a structure of data given a directional character of the data used in system modeling. Experimental studies are reported in Section 4 in which both synthetic and publicly available data sets are considered.

In the paper, we adhere to the standard notation and symbols used commonly in rule-based models and fuzzy sets. In particular, capital letters are used to denote fuzzy sets, while vectors are shown in boldface.

2. A structure of the incremental fuzzy model

We are concerned with the incremental fuzzy model (in essence being a certain rule-based architecture) whose topology is built directly by discovering and exploiting the structure of the data being determined with the use of an augmented model-oriented fuzzy clustering. In the construction of the model we use a collection of data coming in the form of input–output pairs $(\boldsymbol{x}_k,\ target_k)$, $k = 1, 2, \ldots, N$ where $\boldsymbol{x}_k$ is in the n-dimensional input space, $\boldsymbol{x}_k \in \boldsymbol{R}^n$, $target_k \in \boldsymbol{R}$.

The generic global model built for these data comes in the form of a certain input–output relationship $f(\boldsymbol{x})$. Typically, this function is very simple and describes the relationships in a global fashion. For instance $f(\boldsymbol{x})$ could be sought as constant, linear or quadratic (polynomial of the second order), say $f(\boldsymbol{x}) = const$, $f(\boldsymbol{x}) = \alpha_0 + \boldsymbol{\alpha}^T\boldsymbol{x}$ etc. Owing to form of the model, its parameters could be easily estimated by using a standard least square error method for which analytical solutions are obtained. In light of the simple form of the global model (constant or linear), its approximation abilities are limited. To alleviate this shortcoming, we look at the error coming as a result of using the model, where the error ε is defined as the following difference $\varepsilon = target - f(\boldsymbol{x})$, determine its structure especially look at the distribution of error in the input space (error clusters) and on the basis of these clusters construct a collection of rules, which compensate for the observed error. In this way, we form a family of fuzzy clusters A_i in $\boldsymbol{R}^n$ where each of them is associated with its corresponding typical