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Multi-confidence rule acquisition and confidence-preserved attribute reduction in interval-valued decision systems

Xiao Zhang [a], Changlin Mei [a,*], Degang Chen [b], Jinhai Li [c]

[a] Department of Statistics, School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an, PR China
[b] Department of Mathematics and Physics, North China Electric Power University, Beijing, PR China
[c] Faculty of Science, Kunming University of Science and Technology, Kunming, PR China

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ABSTRACT

Rule acquisition is one of the most important objectives in the analysis of decision systems. Because of the interference of errors, a real-world decision system is generally inconsistent, which can lead to the consequence that some rules extracted from the system are not certain but possible rules. In practice, however, the possible rules with high confidence are also useful in making decision. With this consideration, we study how to extract from an interval-valued decision system the compact decision rules whose confidences are not less than a pre-specified threshold. Specifically, by properly defining a binary relation on an interval-valued information system, the concept of interval-valued granular rules is presented for the interval-valued decision system. Then, an index is introduced to measure the confidence of an interval-valued granular rule and an implication relationship is defined between the interval-valued granular rules whose confidences are not less than the threshold. Based on the implication relationship, a confidence-preserved attribute reduction approach is proposed to extract compact decision rules and a combinatorial optimization-based algorithm is developed to compute all the reducts of an interval-valued decision system. Finally, some numerical experiments are conducted to evaluate the performance of the reduction approach and the gain of using the possible rules in making decision.

1. Introduction

It has been recognized that the traditional *rough set theory*, proposed by Pawlak [10], is powerful in discovering knowledge in the information systems and decision systems with nominal attributes [11–14,28,29]. In practice, however, the data collected from practical problems are usually real numbers and generally include errors due to the human cognitive uncertainty and/or interference of some random factors. Therefore, an information system with its attribute values being intervals is perhaps more appropriate for describing such data because one of the useful ways for characterizing the value of a variable with uncertainty is to use the interval specified by the properly defined lower and upper limits of the values that this variable possibly takes. Furthermore, from the viewpoint of rule acquisition, as pointed out in [6], although the decision rules derived directly from a decision system with real-valued conditional attributes may be accurate or certain, their generalization ability will most likely be rather low because perfect match of the real conditional attribute values is in

* Corresponding author.
E-mail addresses: zhangxiaoo@126.com (X. Zhang), clmei@mail.xjtu.edu.cn (C. Mei), wscz973@163.com (D. Chen), jhlixjtu@163.com (J. Li).

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general impossible; however, if the conditional attribute values are intervals, the induced decision rules are of more powerful generalization ability. Therefore, it is worth studying to extract decision rules from an interval-valued decision system because of its wide application backgrounds.

On the other hand, due to the impact of uncertainty in data, a decision system is generally inconsistent, which results in the fact that some or even most of the decision rules derived from the decision system are not certain but possible rules. In this situation, it is usually a common practice that one makes decision according to the confidence degree of the possible rules. Therefore, besides the certain rules derived from a decision system, some uncertain or possible rules with high confidence are also useful in making decision. Along this line of thinking, extracting from a decision system not only the certain rules but also the possible rules with a high confidence level is of importance in practice. Nevertheless, for interval-valued decision systems, how to extract both certain rules and the possible rules with high confidence remains to be studied. Furthermore, in order to make the extracted rules compact, the related attribute reduction issue also deserves to be investigated.

In this paper, we develop a multi-confidence rule acquisition framework for interval-valued decision systems to extract the decision rules whose confidences are not less than a pre-specified threshold. Furthermore, a confidence-preserved attribute reduction procedure is proposed for acquiring compact or concise multi-confidence rules. Specifically, we first define in an interval-valued information system a binary relation and study some basic properties of the lower and upper approximation operators based on the binary relation. Then, the concept of interval-valued granular rules is presented for the interval-valued decision system and the confidence index of a decision rule introduced by Kryszkiewicz [5] is extended to the interval-valued decision systems for measuring the confidence of an interval-valued granular rule in interval-valued decision systems. Furthermore, we define a proper implication relationship between the interval-valued granular rules whose confidences are not less than a pre-specified threshold. Based on this implication relationship, a confidence-preserved attribute reduction framework is formulated and a combinatorial optimization-based algorithm is proposed to compute all the reducts. Finally, some numerical experiments are conducted to assess the performance of the proposed reduction method and the gain of using the possible rules in decision making.

2. The related work

In fact, much attention has recently been paid to interval-valued information systems due to their wide application backgrounds. For example, Leung et al. [6] presented a rough set method for the discovery of the α-classification rules in interval-valued decision systems. Miao et al. [8] developed a new framework of knowledge reduction in interval-valued information systems based on the maximal consistent blocks. Dai et al. [1] studied the uncertainty measurement problem in interval-valued decision systems based on an extended conditional entropy and the notion of the possible degree between intervals. Qian et al. [15] introduced a dominance relation to interval-valued information systems, provided an object-ranking method using the whole dominance degree of each object, and proposed an attribute reduction procedure to extract compact dominance rules. Yang et al. [21] developed a data complement method to transform an incomplete interval-valued information system with a dominance relation into a complete one and proposed six types of relative reducts according to the different requirements of simplifying the dominance rules supported by an object. Based on the grey system theory, Yamaguchi et al. [20] built a new rough set model for the information systems containing interval data, and made an extensive comparison with some other rough approximation models. Yamaguchi et al. [19] investigated the issues of the extraction and the reduction of the decision rules based on grey lattice classification. Gong et al. [2] developed the basic theory of the interval-valued rough fuzzy sets by combining interval-valued fuzzy sets with the rough sets. Sun et al. [17] presented a rough approximation of an interval-valued fuzzy set and investigated the issue of attribute reduction in the interval-valued fuzzy information system. Motivated by the need of error analysis in numeric computation, Moore [9] put forward an interval-number algebra by extending the elementary arithmetic to the interval arithmetic. As a counterpart of the interval-number algebra, Yao [22] introduced an interval-set algebra for representing qualitative information. Yao and Li [23] compared rough set and interval set models and showed that these two models provide different and complementary extensions of the set theory. Some interval approaches for uncertainty reasoning were proposed in [24]. Yao and Liu [25] presented the basic formulation and interpretation of the generalized decision logic in interval-set-valued information tables. Yao [26] made a comprehensive review on interval sets and interval-set algebras. By using an interval set model, Li et al. [7] proposed a novel method for extracting rules from an incomplete decision system. Zhang and Su [27] presented a monotonic inclusion measure approach to rank two intervals. Although interval-valued information (decision) systems have been studied from different perspectives, the issue of extracting both certain and possible rules for making decision, as mentioned in Section 1, remains to be investigated and is the main topic that this paper concerns.

3. Preliminaries

3.1. Traditional rough sets and decision systems

An information system is a pair (U, A) with a mapping $a(x) : U \times A \to V_a$ for each pair $(x, a) \in U \times A$, where U is a non-empty finite set of objects called the universe of discourse, A is a non-empty finite set of attributes, and V_a is the domain of the attribute a.