International Journal of Approximate Reasoning 55 (2014) 1129–1146

Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

www.elsevier.com/locate/ijar

Decomposition of conflict as a distribution on hypotheses in the framework on belief functions

Arnaud Roquel [a], Sylvie Le Hégarat-Mascle [a,*], Isabelle Bloch [b], Bastien Vincke [a]

[a] Institut d'Electronique Fondamentale, Univ. Paris-Sud/CNRS, 91405 Orsay, France
[b] Institut Mines-Telecom, Telecom ParisTech, CNRS LTCI, Paris, France

ARTICLE INFO

Article history:
Received 13 June 2013
Received in revised form 22 December 2013
Accepted 23 December 2013
Available online 31 December 2013

Keywords:
Belief functions
Conflict
Conflict decomposition
Distribution of conflict over hypotheses
Vehicle localization

ABSTRACT

In this paper, we address the problem of identifying the potential sources of conflict between information sources in the framework of belief function theory. To this aim, we propose a decomposition of the global measure of conflict as a function defined over the power set of the discernment frame. This decomposition, which associates a part of the conflict to some hypotheses, allows identifying the origin of conflict, which is hence considered as “local” to some hypotheses. This is more informative than usual global measures of conflict or disagreement between sources. Having shown the unicity of this decomposition, we illustrate its use on two examples. The first one is a toy example where the fact that conflict is mainly brought by one hypothesis allows identifying its origin. The second example is a real application, namely robot localization, where we show that focusing the conflict measure on the “favored” hypothesis (the one that would be decided) helps us to robustify the fusion process.

1. Introduction

Multi-sensor systems are used in many applications such as classification, image processing, change detection, object trajectory localization. Usually the information provided by each sensor is prone to imperfections, such as imprecision and uncertainty, and fusion procedures aim at making better decisions by combining multi-sensor information. Belief Functions (BF) are suitable for modeling imprecision and uncertainty, and handle belief on the power set of the frame of discernment (set of hypotheses).

A disagreement between sources indicates a possibly unreliable result from which may derive a bad decision. Thus, for managing the disagreement, several authors have developed different combination rules where “Dempster's conflict” [48] is transferred to a set of elements. There are two main ways to redistribute Dempster's conflict: Either globally, using Dempster's rule [45] or Yager's rule [52], or based on a conflict estimation local to a considered hypothesis, for instance using Dubois and Prade's rule [12], which can be considered as a redistribution rule (although not initially intended as such), as shown in [21], or using a Proportional Conflict Redistribution rule (e.g. PCR5 [47]). In [25,28], the conflict is estimated locally to a hypothesis and a source, and relatively to the other hypotheses supported by the other sources. This local estimation is assumed to be a local indicator of source “reliability”, and then used to automatically discount the source. Florea et al. [14] proposed a more robust (according to the authors) combination that is a weighted sum of conjunctive and disjunctive rules, with the weights being functions of Dempster's conflict.

A few works have proposed to use the conflict in other tasks than the one of combination. For instance, in [37,38], Dempster's conflict was used as a precision criterion of superimposition of the images provided by the different sources. The authors in [35] proposed to use the conflict between a model predicting the evolution of a system state and observations

* Corresponding author.

0888-613X/$ – see front matter
http://dx.doi.org/10.1016/j.ijar.2013.12.006

to decide whether the model was appropriate or not, and then potentially change the model. Schubert [41,42] proposed to compute pairwise conflicts between sources, to use the conflict values for clustering the sources, and to combine only non-conflicting sources (grouped into the same cluster). In the same spirit, Klein and Colot [19] defined a series of measures of conflict that is function of discounting rates and based singular source mining on it. In [43], a measure involving both Dempster's conflict and the aggregated uncertainty [24,16,29,17] was used to evaluate the relevance of different discernment frames.

Finally, besides these studies considering the conflict as a potential information source, several works focused on the estimation of the conflict itself. A review can be found in [22], where the author argues that two beliefs in conflict should present both a high Dempster's conflict value and a high distance value. Indeed, the dissimilarity between beliefs is often measured by a distance (see [18] for a review of the different distance measures between beliefs). A typical example is to compute an L_p-norm distance between mass functions. In [27], Jousselme's distance was used to evaluate the conflict introduced by a belief into a set of beliefs. Now, authors generally agree that a distance is not a direct measure of conflict. Recently, some works have proposed alternative definitions to Dempster's conflict, e.g. [9,26,43,44,11]. Anyway, Dempster's conflict remains a strong indicator, even if its interpretation may not be so straightforward.

In this paper, that extends our preliminary work in [40], we aim at a finer analysis of source disagreement based on Dempster's conflict. In particular, we propose a decomposition of Dempster's conflict, which is related to the different elements of the frame of discernment and thus helps identifying the potential link between the conflict and particular hypotheses. It only applies for non-dogmatic belief.

The proposed decomposition can be used to analyze the intra-source conflict (i.e. the conflict inherent to a source when modeling the information it delivers as belief functions) or the conflict between sources (i.e. the conflict which appears when merging sources). This decomposition could also be used to design new combination rules. This step is out of the scope of this paper and we focus on the decomposition and its analysis.

After reminding some notations and basic elements on mass function decompositions and distance measures in Section 2, the proposed function of conflict is introduced and analyzed in Section 3, while numerical aspects are detailed in Section 4. It is then illustrated on a vehicle localization problem in Section 5. Conclusions are provided in Section 6.

2. Background

In the following, we denote by Ω the frame of discernment and by 2^Ω the power set of Ω. A Basic Belief Assignment (BBA) on 2^Ω, noted m and also called mass function, is a function from 2^Ω into $[0,1]$ such that $\sum_{A\in 2^\Omega} m(A) = 1$. The elements A of 2^Ω with a non-null mass value, i.e. $m(A) > 0$, are called the focal elements of the BBA. Three specific kinds of BBAs are:

- the BBAs, called "non-dogmatic", that have Ω as focal element,
- the BBAs having only two focal elements, one of which being Ω, called "Simple Support Functions (SSF)" when referring to the canonical decomposition (presented further),
- the BBAs, called "consonant", that have nested focal elements, i.e. $\forall A \subseteq \Omega \mid m(A) > 0$, $\forall B \subseteq \Omega \mid m(B) > 0$, $A \subseteq B$ or $B \subseteq A$.

Plausibility and commonality are denoted by Pl and q, respectively. They are functions on 2^Ω: $\forall A \in 2^\Omega$, $Pl(A) = \sum_{B\cap A\neq\emptyset} m(B)$, $q(A) = \sum_{B\supseteq A} m(B)$. Note that since the functions m, Pl and q (as the two other belief functions not used in this work, namely the credibility Bel and the implicability b) are in one to one correspondence, they represent the same belief.

The dissimilarity between BBAs is often used for computing the disagreement between the corresponding sources. As said in the Introduction, it is generally estimated from a conflict or distance measure (see e.g. [18,22]). Specifically, "Dempster's conflict" is computed, in the case of two BBAs m_1 and m_2, as follows:

$$K_{1,2} = \sum_{A\subseteq\Omega} \sum_{\substack{B\subseteq\Omega \\ A\cap B=\emptyset}} m_1(A) m_2(B). \tag{1}$$

When several beliefs are available, combining them allows deriving a global belief. Specifically, the orthogonal sum was defined by Dempster [45] to combine, in a conjunctive way, N BBAs assumed to be independent. It involves a normalization term that corresponds to the assumption of a closed world[1] and writes, in the case of two BBAs m_1 and m_2, $1 - K_{1,2}$ (Eq. (1)). Alternatively, assuming an open world,[2] Smets proposed [49] to remove the normalization and to assign the value $K_{1,2}$ to the mass of the empty set. Then, Smets' combination [49] (sometimes simply called conjunctive rule because of its authority) writes:

[1] Exhaustive discernment frame.

[2] Non-exhaustive discernment frame.