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Possibilistic sequential decision making

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ABSTRACT

When the information about uncertainty cannot be quantified in a simple, probabilistic way, the topic of possibilistic decision theory is often a natural one to consider. The development of possibilistic decision theory has lead to the proposition a series of possibilistic criteria, namely: optimistic and pessimistic possibilistic qualitative criteria [7], possibilistic likely dominance [2,9], binary possibilistic utility [11] and possibilistic Choquet integrals [24]. This paper focuses on sequential decision making in possibilistic decision trees. It proposes a theoretical study on the complexity of the problem of finding an optimal strategy depending on the monotonicity property of the optimization criteria – when the criterion is transitive, this property indeed allows a polytime solving of the problem by Dynamic Programming. We show that most possibilistic decision criteria, but possibilistic Choquet integrals, satisfy monotonicity and that the corresponding optimization problems can be solved in polynomial time by Dynamic Programming. Concerning the possibilistic likely dominance criteria which is quasi-transitive but not fully transitive, we propose an extended version of Dynamic Programming which remains polynomial in the size of the decision tree. We also show that for the particular case of possibilistic Choquet integrals, the problem of finding an optimal strategy is NP-hard. It can be solved by a Branch and Bound algorithm. Experiments show that even not necessarily optimal, the strategies built by Dynamic Programming are generally very good.

1. Introduction

For several decades, there has been a growing interest in Operation Research and more recently in Artificial Intelligence towards the foundations and computational methods of decision making under uncertainty. This is especially relevant for applications to sequential decision making under uncertainty, where a suitable strategy is to be found, that associates a decision to each state of the world. Several representation formalisms can be used for sequential decision problems, such as decision trees, influence diagrams or Markov decision processes. A decision tree is an explicit representation of a sequential decision problem, while influence diagrams or Markov decision processes are compact representations. In this paper, we focus on the former framework: even in this simple, explicit, case, the set of potential strategies is combinatorial (i.e., its size increases exponentially with the size of the tree); the determination of an optimal strategy for a given representation and a given decision criterion is then an algorithmic issue in itself.

A popular criterion to compare decisions under risk is the expected utility (*EU*) model axiomatized by Von Neumann and Morgenstern [19]. This model relies on a probabilistic representation of uncertainty: an elementary decision (i.e. a one-step decision problem) is modeled by a probabilistic lottery over the possible outcomes. The preferences of the decision maker are supposed to be captured by a utility function assigning a numerical value to each outcome. The evaluation of a lottery is

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then performed through the computation of its expected utility (the greater, the better). In sequential decision making, each possible strategy is viewed as a compound lottery. It can be reduced to an equivalent simple lottery, and thus compared to remaining ones according to its expected utility.

Operational Research then proposes an efficient tool for the optimization of expected utility in probabilistic decision trees: *Dynamic Programming*. Although the high combinatorial nature of the set of possible strategies, the selection of an optimal strategy can be performed in time polynomial with the size of the decision tree: the EU model indeed satisfies a property of monotonicity that guarantees the completeness of Dynamic Programming.

When the information about uncertainty cannot be quantified in a probabilistic way the topic of possibilistic decision theory is often a natural one to consider [2,4,7]. Giving up the probabilistic quantification of uncertainty yielded to give up the EU criterion as well. The development of possibilistic decision theory has lead to the proposition and often of the characterization of a series of possibilistic counterparts of the EU criterion. Rebille [24], for instance, advocates the use of possibilistic Choquet integrals, which relies on a numerical interpretation of both possibility and utility degrees. On the contrary, Dubois and Prade [7] have studied the case of a qualitative interpretation and propose two criteria based on possibility theory, an optimistic and a pessimistic one (denoted U_{opt} and U_{pes}), whose definitions only require a finite ordinal, non-compensatory, scale for evaluating both utility and plausibility.

The axiomatization of U_{opt} and U_{pes} has given rise to the development of sophisticated qualitative models for sequential decision making, e.g. possibilistic Markov Decision Processes [25,26], possibilistic ordinal Decision Trees [10] and even possibilistic ordinal Influence Diagrams [14]. One of the most interesting properties of this qualitative model is indeed that it obeys a weak form of the monotonicity property. As a consequence, Dynamic Programming may be used and an optimal strategy with respect to U_{opt} or U_{pes} can be built in polytime, just like in the case of expected utility.

On the contrary, general Choquet integrals are incompatible with Dynamic Programming. Worst, the problem of determining an optimal strategy with respect to Choquet integrals is NP-hard in the general case [15]. We will show in the present paper that the problem of determining a strategy optimal with respect to a *possibilistic* Choquet integrals is NP-hard as well.

More generally, this paper gives a deep study of complexity of strategy optimization problem w.r.t. possibilistic decision criteria and proposes a resolution algorithm (Dynamic Programming or Branch and Bound) for each criterion according to its complexity class (*P* or *NP*).

This paper[1] is organized as follows: Section 2 presents a refresher on possibilistic decision making under uncertainty and a short survey on most common possibilistic decision criteria. Section 3 then presents our results about the complexity of sequential decision making in possibilistic decision trees. Finally, Section 4 is devoted to the proposition of a Branch and Bound algorithm for the optimization of Choquet-based possibilistic decision trees in the general case. For the sake of readability, the proofs have been gathered in Appendix A.

2. Possibilistic decision theory

2.1. Basics of possibility theory

Possibility theory, issued from Fuzzy Sets theory, was introduced by Zadeh [31] and further developed by Dubois and Prade [5]. This subsection gives some basic elements of this theory, for more details see [5].

The basic building block in possibility theory is the notion of *possibility distribution* [5]. Let $X_1, \dots, X_n$ be a set of state variables whose value are ill-known such that $D_1, \dots, D_n$ are their respective domains. $\Omega = D_1 \times \dots \times D_n$ denotes the universe of discourse, which is the cartesian product of all variable domains in $X_1, \dots, X_n$. Vectors $\omega \in \Omega$ are often called realizations or simply "states" (of the world). The agent's knowledge about the value of the x_i's can be encoded by a possibility distribution $\pi : \Omega \to [0, 1]$; $\pi(\omega) = 1$ means that realization ω is totally possible and $\pi(\omega) = 0$ means that ω is an impossible state. It is generally assumed that there exist at least one state ω which is totally possible – π is said then to be *normalized*.

Extreme cases of knowledge are presented by:

- *complete knowledge*, i.e. $\exists \omega_0$ s.t. $\pi(\omega_0) = 1$ and $\forall \omega \neq \omega_0$, $\pi(\omega) = 0$,
- *total ignorance*, i.e. $\forall \omega \in \Omega$, $\pi(\omega) = 1$ (all values in Ω are possible).

From π, one can compute the possibility $\Pi(A)$ and the necessity $N(A)$ of an event $A \subseteq \Omega$:

$$\Pi(A) = \sup_{\omega \in A} \pi(\omega), \tag{1}$$

$$N(A) = 1 - \Pi(\bar{A}) = 1 - \sup_{\omega \notin A} \pi(\omega). \tag{2}$$

Measure $\Pi(A)$ evaluates to which extend A is *consistent* with the knowledge represented by π while $N(A)$ corresponds to the extent to which $\neg A$ is impossible and thus evaluates at which level A is certainly implied by the knowledge.

[1] This paper is an extended version of a preliminary work about the complexity of possibilistic decision trees presented in [8].