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Triangular fuzzy decision-theoretic rough sets

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ABSTRACT

Based on decision-theoretic rough sets (DTRS), we augment the existing model by introducing into the granular values. More specifically, we generalize a concept of the precise value of loss function to triangular fuzzy decision-theoretic rough sets (TFDTRS). Firstly, ranking the expected loss with triangular fuzzy number is analyzed. In light of Bayesian decision procedure, we calculate three thresholds and derive decision rules. The relationship between the values of the thresholds and the risk attitude index of decision maker presented in the ranking function is analyzed. With the aid of multiple attribute group decision making, we design an algorithm to determine the values of losses used in TFDTRS. It is achieved with the use of particle swarm optimization. Our study provides a solution in the aspect of determining the value of loss function of DTRS and extends its range of applications. Finally, an example is presented to elaborate on the performance of the TFDTRS model.

1. Introduction

Rough set theory is a new mathematical tool to deal with uncertainty problem [41]. As an extension model of original rough sets, probabilistic rough sets (PRS) play a significant role in rough sets and attract the attention of many researchers [57,59]. In recent years, a series of PRS models [57] were proposed, such as 0.5-probabilistic rough sets, decision-theoretic rough sets (DTRS), variable precision rough sets (VPRS), Bayesian rough sets, parameterized rough sets, game-theoretic rough sets (GTRS), probabilistic rough set over two universes, etc. The determination for a pair of thresholds used in PRS becomes a substantial challenge [30]. The pair of thresholds in the most PRS models need a reasonable semantic interpretation [59,61,62]. By introducing game theory into PRS, Herbert and Yao [17] proposed GTRS to determine the values of thresholds used in PRS. Herbert and Yao [18] investigated the GTRS model and its capability of analyzing a major decision problem evident in the existing PRS. Azam and Yao [4] extended GTRS for formulating and analyzing multiple criteria decision making problems in rough sets. Using Shannon entropy as a measure of uncertainty, Deng and Yao [13] presented an information-theoretic approach to the interpretation and determination of thresholds used in PRS. DTRS was proposed by Yao et al. [55,56], which provided a new interpretation in the aspect of determining the threshold values. For the DTRS model, the pair of thresholds presented in PRS can been calculated by loss function with the minimum expected overall risk, where the losses are associated with the decision risk. DTRS has been applied to many domains, such as email filtering [67], investment decision [29,33], cluster [26], text classification [23], information filtering [22], web-based support systems [54], etc. Hence, DTRS has became an important research direction of rough sets.

From the viewpoint of semantics, Yao [63] reviewed several generalized (modified) models and the applications of the DTRS model. In the granulated view of the universe, Abd El-Monsef and Kilany [1] proposed a generalized decision-theoretic model based on an general binary relation. Greco et al. [16] proposed a Bayesian decision theory for dominance-based rough set approach (DRSA), which was permitted to take into account costs of misclassification in variable consistency

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DRSA. According to different attitudes of decision makers, Li and Zhou [24] proposed multi-view decision models of DTRS. Based on the misclassification cost and the test cost, Li et al. [25] designed an algorithm for searching an optimal test set of attributes with the minimum total cost. Liu et al. [31] and Lingras et al. [26] extended DTRS with two-category to multi-category. Considering the losses with probabilistic distribution, Liu et al. [32] proposed an extension of DTRS under the uniform and the normal distribution. Liu et al. [34] designed a method for estimating the conditional probability using logistic regression. Ma and Sun [35] extended Bayesian risk decision of PRS of the same universe to two universes. Yao [59] used the relative values between loss functions to express the thresholds. Yao and Zhou [60] proposed a naive Bayesian DTRS, where the conditional probability was estimated by using the Bayes theorem with naive probabilistic independence assumption for attributes. Under the multiple sets of decision preferences and criteria adopted by different agents, Yang and Yao [53] proposed a multi-agent DTRS model. In addition, the attribute reduction of DTRS has been discussed in [20,58,65,66]. With respect to the above discussions, a critical issue of the DTRS model is assigning the loss function.

Mishra et al. [38] found that the fuzzy boundaries implied by vague information could actually help individuals perform better than when being confronted with precise information. In the realistic decision process, some influencing factors also result in decision makers not to provide precise values, e.g., limited domain knowledge of decision maker, tight deadlines, limited budgets. As an extension of precise numerical values, fuzzy set [64] is considered here to deal with vague, imprecise and uncertain problems. These observations form a cornerstone of the model to be developed in this study. Therefore, the value of loss function with the measurement of fuzzy set is more realistic. In the fuzzy set theory, membership function is a basic element. Various approaches of membership function elicitation have been discussed in [15,19,36,37,39,43,48,52]. The membership function elicitation provides a solution to assign the losses of DTRS model. With respect to membership function, triangular fuzzy number is a representative one. A certain theoretically sound motivation behind the common use of triangular membership functions was analyzed in [42]. For simplicity and clarity, we assume the loss function used in DTRS model is a triangular fuzzy number. We focus on constructing triangular fuzzy decision-theoretic rough set (TFDTRS) model. In practical applications, linguistic variable is associated with the (triangular) fuzzy number [12,15,28,39,40,43,47,52]. The scales of linguistic variables with triangular fuzzy numbers usually comply with uniform distribution [28]. Using the information granularity and particle swarm optimization (PSO) [14,46], Pedrycz and Song [43] arrived at a different conclusion. By regulating the scales of linguistic variables, Pedrycz and Song [43,44] proposed a new approach to improve consistency. The approach evolves the negotiation process of experts and can automatically obtain a consistency result. Inspired by [43,44], we further construct a multiple attribute group decision making (MAGDM) to determine the values of losses in the context of TFDTRS. Take example for linguistic variable with triangular fuzzy number, we determine the value of the losses used in TFDTRS. The main contribution of this study can be stated as follows: (a) we provide a method to determine the losses of TFDTRS; (b) we construct a general TFDTRS model to adapt a fuzzy scenario.

The remainder of this paper is organized as follows: Section 2 provides basic concepts of DTRS and triangular fuzzy number. TFDTRS model is proposed and its thresholds are analyzed in Section 3. In the frame of MAGDM, the determination for the values of losses with triangular fuzzy number is designed in Section 4. Then, an example is given to illustrate the application of TFDTRS in Section 5. Section 6 concludes the study and elaborates on future studies.

2. Preliminaries

In this section, basic concepts of decision-theoretic rough sets and triangular fuzzy number are briefly reviewed.

2.1. Decision-theoretic rough sets (DTRS)

Based on the Bayesian decision procedure, the DTRS model is composed of 2 states and 3 actions [59,60]. The set of states is given by $\Omega = \{C, \neg C\}$ indicating that an object is in C and not in C, respectively. The set of actions is given by $\mathcal{A} = \{a_P, a_B, a_N\}$, where a_P, a_B, and a_N represent three actions when classifying object x, namely, deciding $x \in \mathrm{POS}(C)$, deciding x should be further investigated $x \in \mathrm{BND}(C)$, and deciding $x \in \mathrm{NEG}(C)$, respectively. The loss function regarding the risk or cost of actions in different states is given in Table 1.

In Table 1, λ_{PP}, λ_{BP} and λ_{NP} denote the losses incurred for taking actions of a_P, a_B and a_N, respectively, when an object belongs to C. Similarly, λ_{PN}, λ_{BN} and λ_{NN} denote the losses incurred for taking the same actions when the object belongs to $\neg C$. $Pr(C|[x])$ is the conditional probability of an object x belonging to C given that the object is described by its equivalence class $[x]$. For an object x, the expected loss $R(a_i|[x])$ associated with taking the individual action can be expressed as:

Table 1
The loss function regarding the risk or cost of actions in different states.

	C (P)	$\neg C$ (N)
a_P	λ_{PP}	λ_{PN}
a_B	λ_{BP}	λ_{BN}
a_N	λ_{NP}	λ_{NN}